

Inverse functions

Thursday, 19 April 2018 9:34 pm

★ Work to be completed once the teaching has taken place:

5G: Inverse Functions 198 1ac 2abe 3bde 4 7

RECAP:

We have already talked extensively about functions. We have covered:

- What a function is
- How to write a function using function notation
- Domains
- Ranges
- Real Numbers and Integers
- Vertical line tests (to test for a function)
- How to restrict a function
- One to One functions, and
- Hybrid functions

$\Rightarrow f: [0, 4) \rightarrow \mathbb{R}, f(x) = x + 3$

Domain $\uparrow$ new function $\uparrow$

Inverse functions

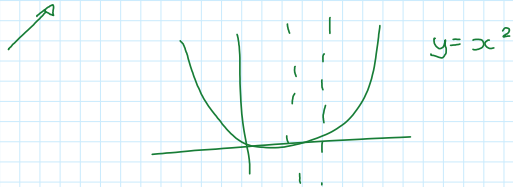
Above all else it is important to note that a **function** can ONLY have an inverse if it is **one to one**.

→ All **relations** can have inverses.

The above is a trick used on SACs all the time!

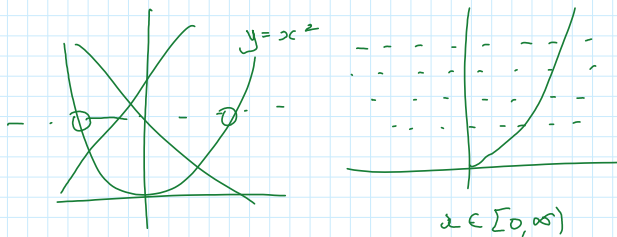
RECAP: Functions

A function is one where a relation can have a **vertical line** pass through it at only one point.
i.e. there are not two y-values for an x-value.



RECAP: One to One Functions

A one-to-one function is a function which, when we draw a horizontal line through it, will only have one point of intersection.
i.e. for any y-value, there will only ever be one x-value.



Notation:
We denote the inverse of a function as $f^{-1}(x)$
This is NOT the same as $(f(x))^{-1}$

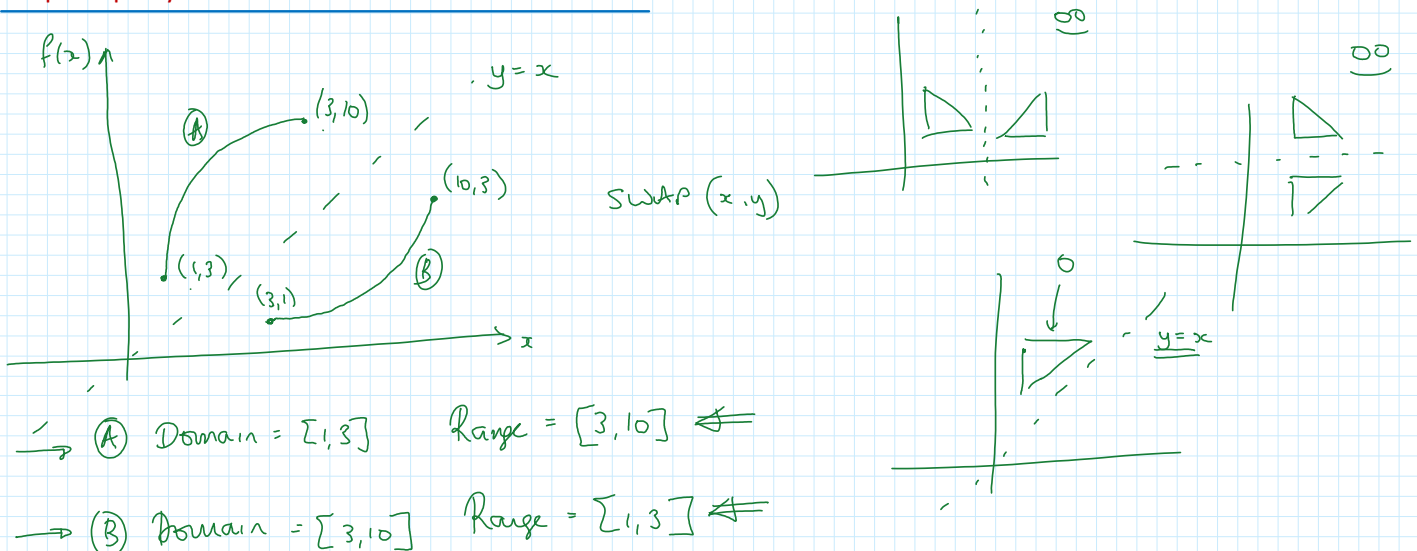
$$\boxed{f^{-1}(x)} = \frac{1}{f(x)}$$

An inverse function is where the original function is reflected in the line $y = x$



REALLY IMPORTANT!

Examples: Graphically



→ ③ Domain = $[3, 10]$ Range = $[1, 3]$ ✗

HINT: Find domain & range ORIGINAL Function!
Inverse ... swap!

Finding the domain and range of functions and their inverses

The domain of an inverse function is equal to the range of the function
The range of an inverse function is equal to the domain of the function



Super IMPORTANT!

Example:

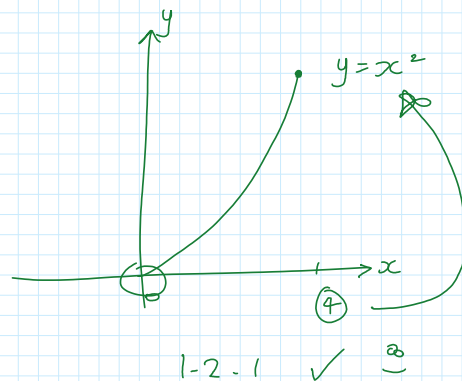
Find the domain and range of the function $f: [0, 4] \rightarrow \mathbb{R}, f(x) = x^2$
Hence, find the domain and range of the inverse function.

Domain = $[0, 4]$

Range = $[0, 16]$ ✗

∴ Domain $f^{-1}(x) = [0, 16]$

Range $f^{-1}(x) = [0, 4]$ ✓✓



Finding the inverse function using algebra.

Example: Find the inverse of the function $f: [0, 4] \rightarrow \mathbb{R}, f(x) = x^2$ ←

Domain = $[0, 16]$ ✗
Range = $[0, 4]$

$f(x) = x^2$

$y = x^2$ ←

$x = y^{(2)}$

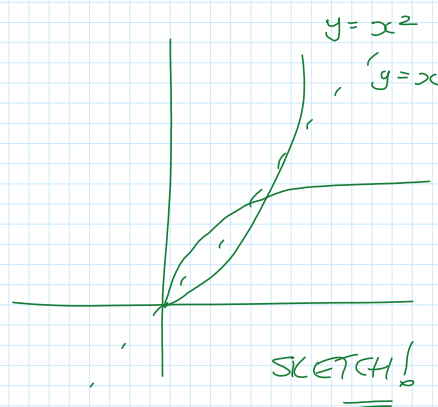
$(\pm)\sqrt{x} = y$ ←

$y = \sqrt{x}$

~~$y = -\sqrt{x}$~~



$f^{-1}: [0, 16] \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt{x}$



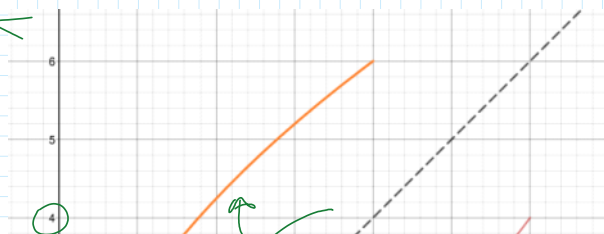
Example: let $f: [3, 6] \rightarrow \mathbb{R}, f(x) = \left(\frac{x}{3}\right)^2$. Find f^{-1} and state its domain and range.

Domain = $[3, 6]$ ←

Range = $[1, 4]$

$\left(\frac{3}{3}\right)^2$

$\left(\frac{6}{3}\right)^2$



$$\text{Range} = [1, 4] \quad (3) \quad (3)$$

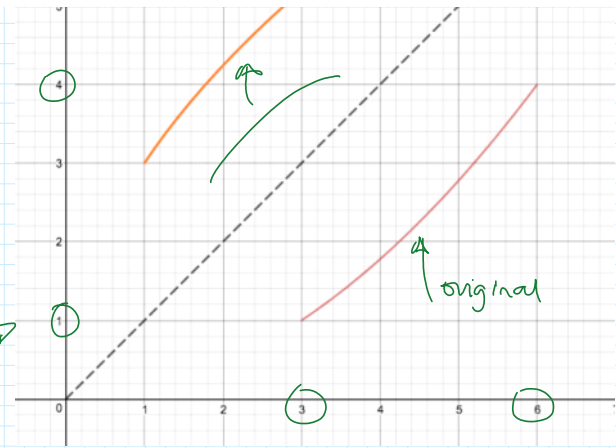
$$f^{-1}(x) \quad \text{Domain} = [1, 4] \quad (1) \quad \checkmark$$

$$\text{Range} = [3, 6]$$

$$f(x) = \left(\frac{x}{3}\right)^2 \Rightarrow y = \left(\frac{x}{3}\right)^2 \quad \pm 3\sqrt{x}$$

$$x = \left(\frac{y}{3}\right)^2 \quad \pm 3\sqrt{x} \quad -3\sqrt{x} \Rightarrow$$

$$\pm \sqrt{x} = \frac{y}{3}$$



$$f^{-1}: [1, 4] \rightarrow \mathbb{R}, f^{-1}(x) = 3\sqrt{x}$$

Shortcuts (or the recipe) to getting these questions correct all the time!

1. Always make sure the function is one to one
 - a. If it isn't one-to-one you will need to restrict the domain
2. Sketch the graph (using your calculator if you need to).
3. Use the domain to substitute into $f(x)$ and find the range of the function
4. Swap the pronumerals x and y in the original function
 - a. Then rearrange to have $y=$
5. Swap the Domain and Range.
6. Write the function notation for the inverse.

$$x \in [1, 4]$$

$$f(x) \in [3, 6]$$

