

# Highest Common Factor and Lowest Common Multiple

Wednesday, 21 March 2018 6:25 pm

By the end of teaching all students will be asked to complete the following work:

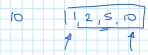
HCF ✓  
LCM ✓

Year 7 Textbook  
Chapter 3  
Exercise 3B  
Questions: 1, 3, 5, 6, 7, 8, 9, 10, 11

## Some important terminology

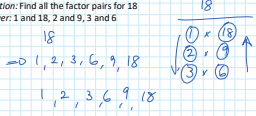
**FACTOR:** A factor is a number which will divide into another number, a whole number of times.

Example:  
Question: List all the factors of 10  
Answer: 1, 2, 5, 10

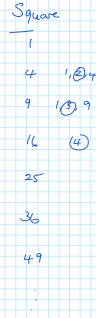
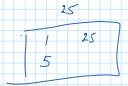
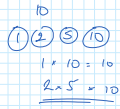


**FACTOR PAIRS:** A pair of factors which, when multiplied together, will give you the original number

Example:  
Question: Find all the factor pairs for 18  
Answer: 1 and 18, 2 and 9, 3 and 6



This is a great way to also find the factors for a number. It makes sure you don't miss one! Hence, we can see the factors are also 1, 2, 3, 6, 9, 18



**MULTIPLE:** A number which is part of a particular multiplication table.

Example:  
Question: Find the first 5 multiple of 2  
Answer: 2, 4, 6, 8, 10

You are really just listing the first 5 numbers in the two times table.

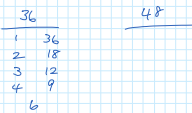
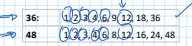
## Highest Common Factor

To find the highest **COMMON** factor we need to be able to **COMPARE** two numbers. If we look at the wording we are looking at **HIGHEST** factor which is **COMMON** between two numbers.

Example:  
Question: Find the highest common factor between 36 and 48  
There are TWO ways of doing this ...

### First way

Find all the factors of each number, and then circle the highest common factor.



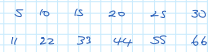
Hence, the highest common factor is 12!

### Second way

This requires us to learn about factor trees. We will do this later.

## Lowest Common Multiple

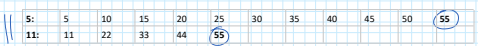
Again, if we look at the wording, it sort of makes sense. We are looking for the lowest number which is in each times table which is shared. There are (again) two ways of doing this!



Example:  
Question: Find the lowest common multiple for 5 and 11

### First way

List all the times tables for each of the numbers and find which is the lowest that they share:



Hence, the lowest common multiple is 55.

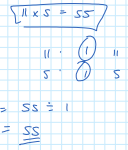
### Second way

The textbook says there is another way of doing this:

If you know the highest common factor for the two numbers, we can use the following fact to find the **Lowest Common Multiple**:

**The lowest common multiple of two numbers can be found by taking the two numbers, multiplying them and then dividing by the Highest Common Factor.**

Example:  
Question: Find the lowest common multiple of 5 and 11  
Firstly, find the highest common factor ...



Factors of 5: 1 and 5  
Factors of 11: 1 and 11  
Highest common factor is 1

Lowest common multiple is  $5 \times 11 \div 1 = 55$

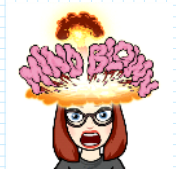
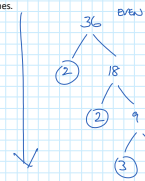
## Another way of doing this

### Prime Factor Trees

#### RECAP: What's a Prime Number?

We can split any number up into a product of its prime numbers!!

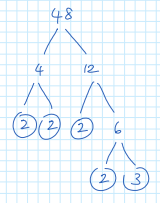
Example:  
Question: Split 36 into the product of its prime numbers  
Remember: **PRODUCT** means times.



$$36 = 2 \times 2 \times 3 \times 3$$

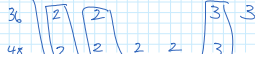
$$36 = 2 \times 2 \times 3 \times 3$$

Now ... look what happens when we do this to the numbers 36 and 48



$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

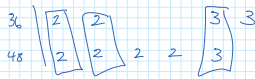
When we compare all the products for the two numbers we see the following:



(2) (3)

When we compare the products for the two numbers we see the following:

36:	2	2			3	3	
48:	2	2	2	2	3		



What does this have to do with Highest Common Factors and Lowest Common Multiples?

Well for the **highest common factor** we look at all the numbers they share as prime factors:

36:	2	2			3	3	
48:	2	2			3	3	

$$HCF = 2 \times 2 \times 3 = 12$$

Which means the Highest Common Factor is  $2 \times 2 \times 3 = 12$

And for the **lowest common multiple** we see that we use one number from each column

36:	2	2			3	3	
48:	2	2	2	2	3		

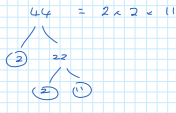
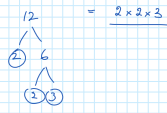
$$LCM = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$$

Which means the Lowest Common Factor is  $2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$

Checking this ...  $36 \times 48 = 1728 \div 12 = 144$

The lowest common multiple of two numbers can be found by taking the two numbers, multiplying them and then dividing by the Highest Common Factor.

Another examples:  
Question: What is the HCF and LCM of 44 and 12



12	2	2	3	1
44	2	2	11	1

$$HCF: 2 \times 2 = 4$$

$$LCM: 2 \times 2 \times 3 \times 11 = 4 \times 3 \times 11 = 132$$

