Graphs of sine and cosine

Wednesday, 28 March 2018 7:21 pm

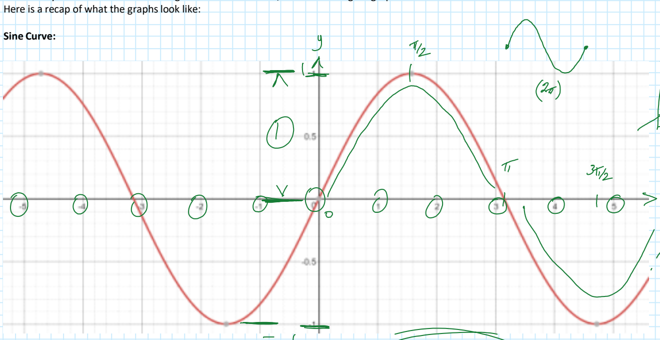
By the end of teaching I would like the following work to be complete:

Graphs of sine and cosine	6D	2,3,4,6,8,10,12	

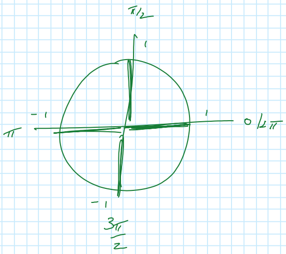
RECAP:

In Year 11 you will have looked at the general form for a sine, cosine and tangent graphs.
Here is a recap of what the graphs look like:

Sine Curve:

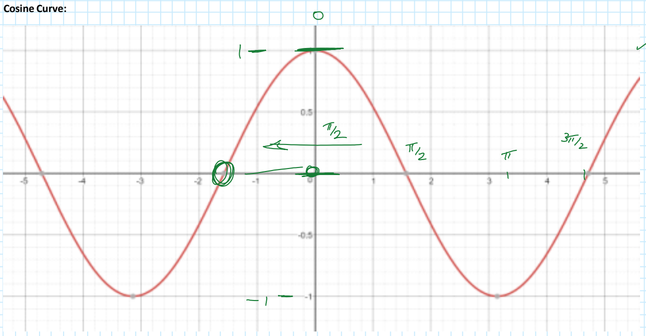


Period = 2π
Maximum $=1$
Minimum$=-1$
Amplitude$=1$



Note: **Period** and **Wavelength** are interchangeable in questions
Note: **Amplitude** is the distance between the mean position and the maximum

Cosine Curve:



Period = 2π
Maximum $=1$
Minimum$=-1$
Amplitude$=1$

Note: The cosine curve is the same as the sine curve which has been translated $\frac{\pi}{2}$ in the negative x direction.

Transformations of sine and cosine curves

Remember that we can transform functions in 3 main ways:

- Dilations
- Reflections
- Translations

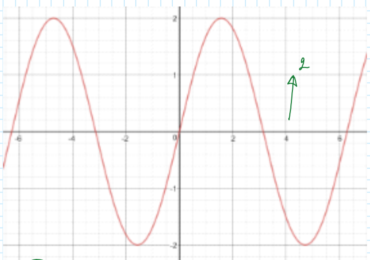
Shortcuts:

Dilations from x-axis: Replace y with $\frac{y}{b}$
Dilations from the y-axis: Replace the x with $\frac{x}{a}$

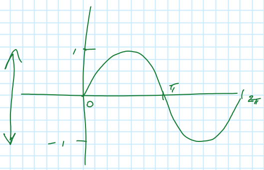
Dilations:

We can dilate in two ways; away from the $x-axis$ and the $y-axis$.
Remember the shortcuts which can be used!!!!

Dilations from x-axis:



Period does not change.
Amplitude = 2

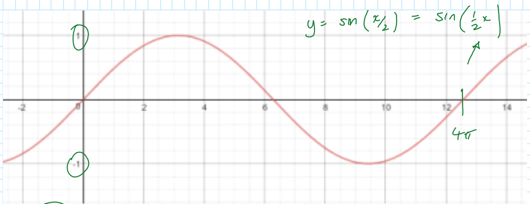


$y = 2sinx$

$f(x) = \sin x$ $af(x) =$

Dilation from the y-axis

$y = \sin(x/2) = \sin(\frac{1}{2}x)$



Amplitude does not change
Period = 4π

Period $= 2\pi \times 2$
$= 4\pi$

Note:
Remember the period of a standard sine curve is 2π

$y = \sin(\frac{x}{2})$

Reflections

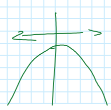
Again, we can use the shortcuts to help us remember, when we have to think of this in terms of algebra.
However, we can reflect in the x-axis and the y-axis.

Shortcuts:

To reflect in the x-axis you replace y with $-y$
To reflect in the y-axis you replace x with $-x$

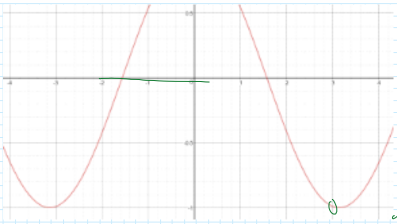
Reflect in the y-axis

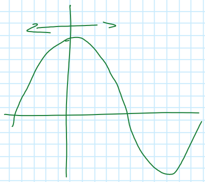
Note: As this is an even function, the reflection has no real effect on the graph



$f(x) \in [-2, 6]$

8

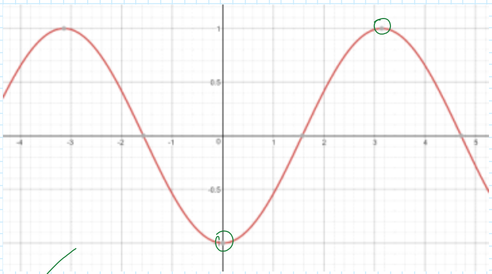


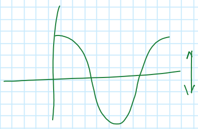


Period = 2π
Amplitude = 1

$y = \cos(-x)$

Reflection in the x-axis

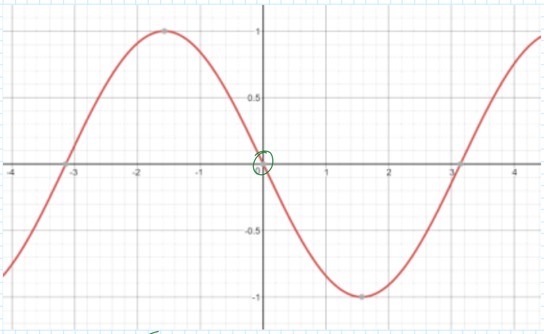




Period = 2π
Amplitude = 1

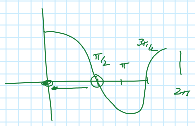
$y = -\cos x$ $\quad y = (-)\cdot\cos x \quad -f(x)$

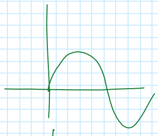
Translations: Horizontal and Vertical

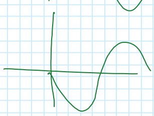


Period = 2π
Amplitude = 1

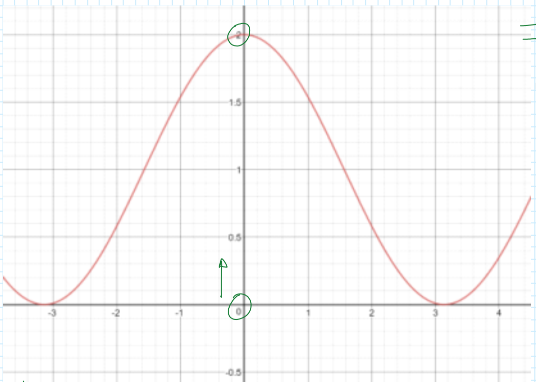
$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$







$y = \cos(x + \frac{\pi}{2})$



Period = 2π
Amplitude = 1

Note: The amplitude is still 1!

$\frac{2-0}{2} = 1$

$\boxed{y = \cos x + 1}$

Graphs of $y = a\sin(nt)$ and $y = a\cos(nt)$

If you notice from the graphs above, we can see that the scales are given in whole numbers.
Even though I have asked for the graphs to be shown in **radians** the scale seems to be whole numbers.
Why?

$y = a\sin(nx)$ multiplier. Amplitude!

$y = \sin nx \quad n = 1$

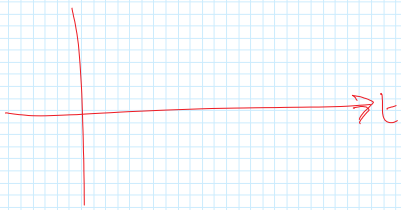
① $y = \sin x \quad$ period = $2\pi \quad \frac{2\pi}{1}$

$y = \sin nx \quad n = \frac{1}{2}$

② $y = \sin\left(\frac{x}{2}\right) \quad$ period = $4\pi \quad \frac{2\pi}{1/2} \quad \frac{2\pi}{(1/2)}$

$y = \sin nx \quad n = 2$

③ $y = \sin(2x) \quad$ period = $\pi \quad \frac{2\pi}{2} \to \frac{\pi}{2} \quad \frac{2\pi}{2}$



$$\boxed{y = \sin(nx) \qquad \text{period} = \boxed{\frac{2\pi}{n}}}$$

Really important Result!

normally expressed

$$\boxed{\begin{array}{ll} y = a\sin(nt) & \text{amplitude} = a \\ y = a\cos(nt) & \text{period} = \dfrac{2\pi}{n} \end{array}}$$

Examples

① Period & Amplitude?

$3\sin^1\theta$ — $A = 3$, Period $= \frac{2\pi}{1} = \underline{2\pi}$

$\frac{1}{2}\sin^1\theta \rightarrow A = \frac{1}{2}$, Period $= 2\pi$

$2\sin\left(\frac{2\theta}{3}\right) \rightarrow A = 2$

Period $= \dfrac{2\pi}{2/3} = 2\pi \div \frac{2}{3} = \cancel{2}\pi \times \frac{3}{\cancel{2}}$

$= \underline{3\pi}$

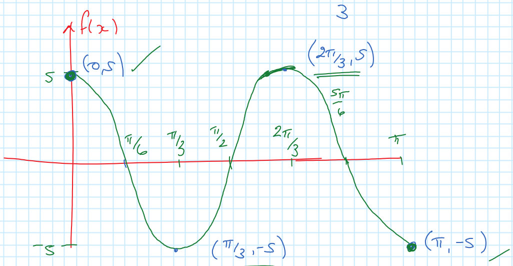
② Sketch $f: \mathbb{R} \rightarrow \mathbb{R},\ f(x) = 5\cos(3x)$ $\underline{0 \leq x \leq \pi}$

Amp $= 5$ Period $= \dfrac{2\pi}{3}$

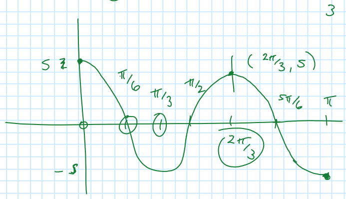
① Always consider the original graph!

② Sketch original and re-calc the crossing points

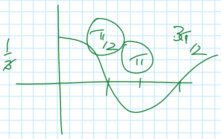
③ Sketch



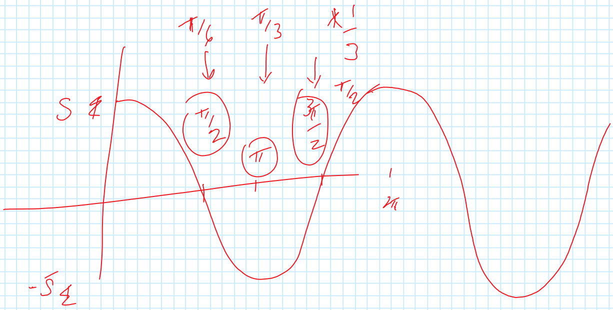
$5\cos 3x$ Period $= \frac{2\pi}{3}$



$\frac{\cancel{3}\pi}{2} \times \frac{1}{\cancel{3}}$



$\frac{\pi}{6} + \frac{4\pi}{6}$ $\frac{\pi}{3} + \frac{2\pi}{3}$ $\frac{3\pi}{3} = \pi$



$\square \div 3$

$\square \times \frac{1}{3}$