Inverse Variation

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to recognise inverse variation.
- To be able to find the constant of variation for inverse variation.
- To be able to solve practical problems involving inverse variation.

Recap

In the last lesson we looked at how two things can be related to each other.

If we have a table of values (or a graph) which when plotted give a straight line, we say that the two quantities are directly proportional to each other.

We can write them in a certain form:

$$a \propto b$$

Which we can then change into an equation by adding the constant of proportionality:

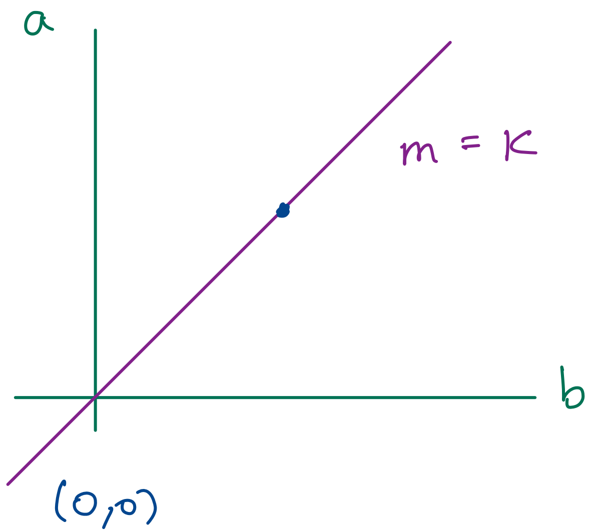
$$a = k \times b$$

This part of the course looks at something called inverse proportionality.

$a \propto b$

$a = k \times b$

$m = \frac{\text{rise}}{\text{run}}$



Inversely proportional to

The graph on the right shows an example of two variables which are inversely proportional to each other. We can see the graph is curved (which a special type of curve).

When two things are inversely proportional to each other we can write them in the following way:

$$y \propto \frac{1}{x}$$

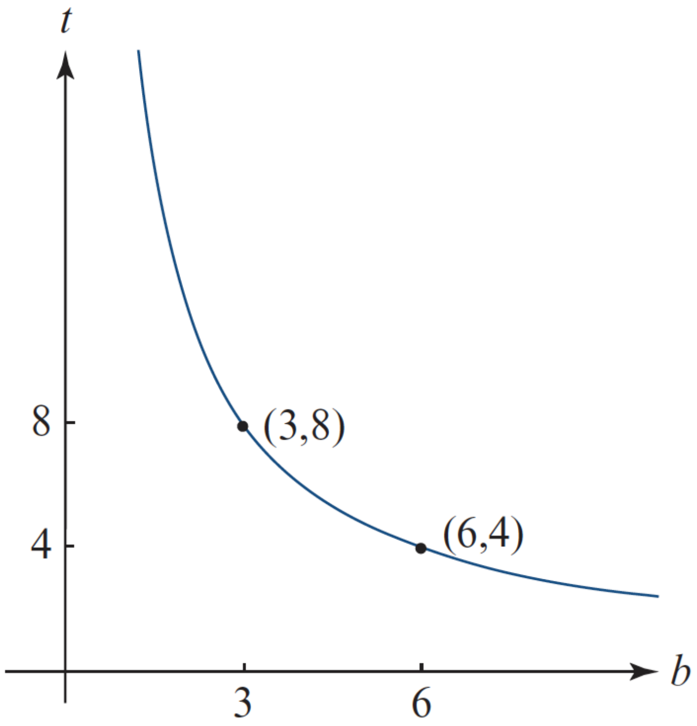
And, using the same idea as the previous lesson, we can change the proportion sign to become:

$$y = k \times \frac{1}{x}$$

$y = mx + c$

$y = k \times \frac{1}{x}$

This can be simplified to become $y = \frac{k}{x}$



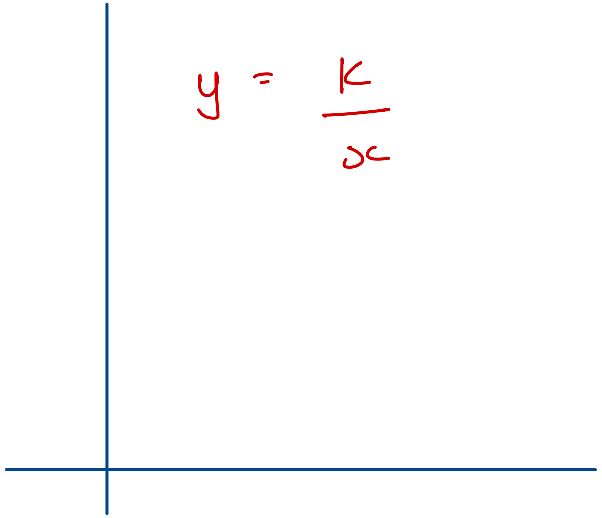
Constant of variation/proportionality

Barry is at it again and now wants to call *k* something else!

So, we need to remember that *k* can be called the **constant of variation** as well as the **constant of proportionality**.

Example. If we know that the following data is inversely proportional to each other, find the value of *k*

x	$\frac{1}{3}$	$\frac{1}{2}$	1	2
y	18	12	6	3



$y = \frac{k}{x}$

$y = \frac{k}{x}$

$18 = \frac{k}{1/3}$

$k = 6$

$\therefore y = \frac{6}{x}$

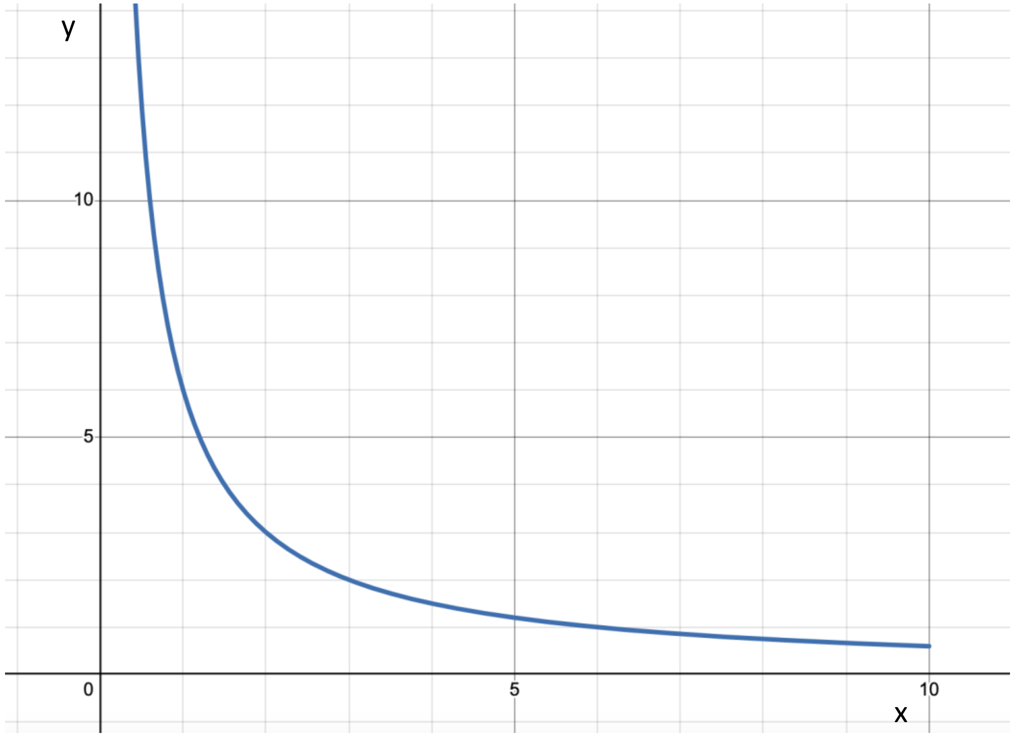
F. $y = \frac{k}{x}$

S. $6 = \frac{k}{1}$

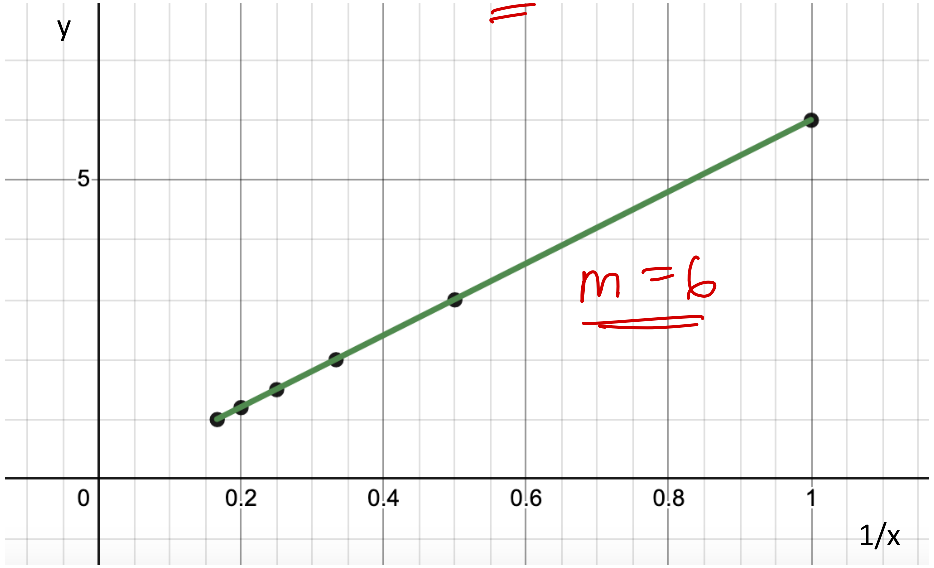
$\therefore k = 6$ ✓

Turning a curve into a straight line

Using the same example as before, when we found that *k=6*, we can draw the original graph and something even more funky!



$k = 6.$



Note that the gradient of the line is 6

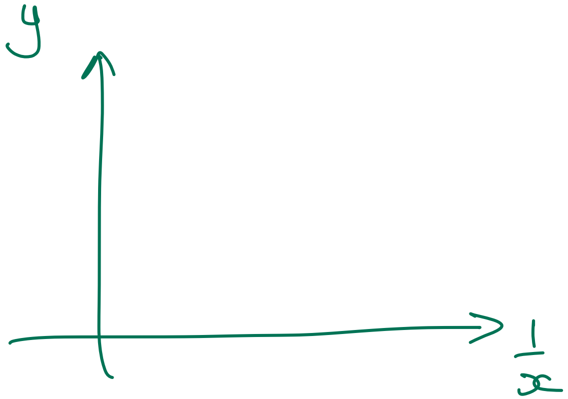
Example

Use the table of values to find the constant of variation, k, and hence complete the table.

$y \propto \dfrac{1}{x}$

x	1	2	3	4	5
y	3	1.5	1	0.75	0.6

$y = k \times \dfrac{1}{x}$



$y = \dfrac{k}{x}$

$0.6 = \dfrac{3}{x}$

$y = \dfrac{k}{x}$

$3 = \dfrac{k}{1}$

$\therefore k = 3$

$\therefore y = \dfrac{3}{x}$

$\therefore y = \dfrac{3}{2}$

Example

Use the table of values to find the constant of variation, k, and hence complete the table.

$$y \propto \frac{1}{x}$$

x	2	4	5	10	20
y	1	0.5	0.4	0.2	0.1

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$\times 2$ $\qquad$ $\times 2$

$$1 = \frac{k}{\cancel{2}}$$

$$\underline{\underline{2 = k}}$$

$$y = \frac{2}{x}$$

$$\therefore y = \frac{2}{10}$$

$$y = 0.2$$

$$y = \frac{2}{x}$$

$$0.1 = \frac{2}{x}$$

$$\underline{\underline{x = 20}}$$

Example

For a cylinder of fixed volume, the height (h cm) is inversely proportional to the square of the radius (r cm).
If a cylinder of height 15 cm has a base radius of 4.2 cm, how high would a cylinder of equivalent volume be if its radius was 3.5 cm?

$$h \propto \frac{1}{r^2}$$

$$h = k \times \frac{1}{r^2}$$

$$h = \frac{k}{r^2}$$

$$15 = \frac{k}{4.2^2}$$

$$k = 264.6$$

$$h = \frac{264.6}{r^2}$$

$$\therefore h = \frac{264.6}{3.5^2}$$

$$h = 21.6 \text{ cm}$$

Example

The time taken (t hours) to empty a tank is inversely proportional to the rate, r, of pumping. If it takes two hours for a pump to empty a tank at a rate of 1200 litres per minute, how long will it take to empty a tank at a rate of 2000 litres per minute?

$$t \propto \frac{1}{r}$$

$$t = k \times \frac{1}{r}$$

$$t = \frac{k}{r}$$

$$120 = \frac{k}{1200}$$

$$k = 144000$$

min

$$\therefore t = \frac{144000}{r}$$

Lit/min

$$t = \frac{144000}{2000}$$

$$t = 72 \text{ mins}$$

Work to complete

The work I am asking to be completed for this topic is shown below.

This is the minimum work which should be completed. The more questions which are answered the better your chance of success in exams. Questions towards the end of the exercises and in the Chapter Review are the best practice you can do.

Questions to complete:
Exercise 9B: 1, 2, 3, 4, 5, 8, 9, 10

Extension:
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