Direct Variation

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to recognise direct variation.
- To be able to find the constant of variation for direct variation.
- To be able to solve practical problems involving direct variation.

Recap

This is the first lesson in this new section of work relating to Variation. This is a new concept but builds on one we have looked at before.

If we were to do a study of age and height, we would expect there to be a link. The older we are, the taller we get. There might be a link between the two. We might say that the two are “proportional”.

When things are proportional, when one thing grows, something else grows. The rate at which is grows is important to us (and, in different lessons, was known as the gradient).

This section of work is going to look at the link between things.

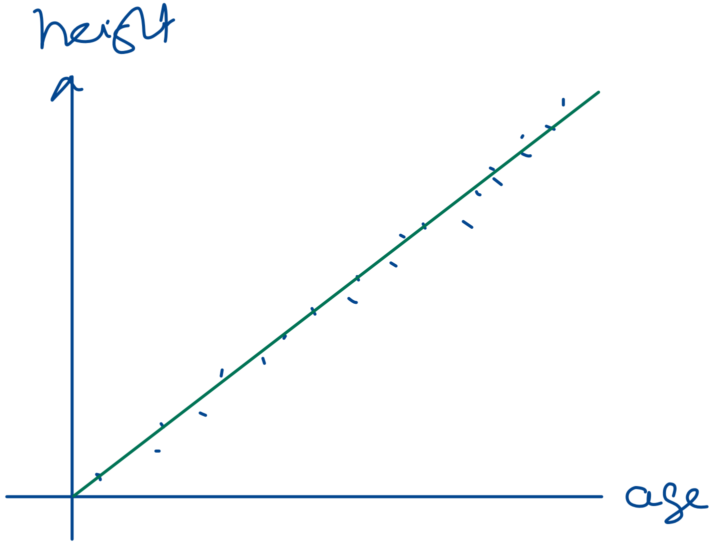
A new symbol: Proportional to

In Mathematics there appears to be a symbol for just about everything!

If we know that height is proportional to age, we can write it in the following way:

$$height \propto age$$

This is the **proportional to** sign.
It can also mean "varies as"



There are other things in life which are proportional to each other.

For example:

- Time and distance travelled in my car
- Time spent studying for a test and test score
- Scores in English and Maths exams
- English comprehension and Mathematics scores.

Changing proportion to equals sign

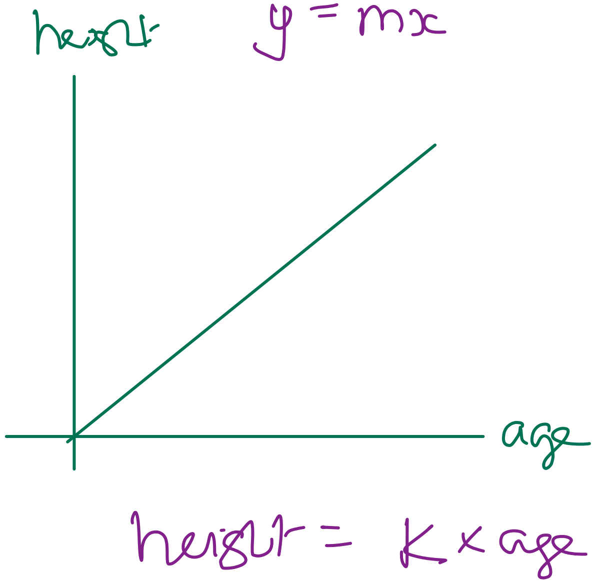
We can't really do anything with something written with a proportion sign. So, we have a way of turning that into an equals sign.

We know that, generally, we can multiply someone's age by a **number** and get their height.

This **number** is called the **constant of proportionality**.

So, we can change the proportion sign ∝ to =k

$$height \propto age$$
$$height = k \times age$$



In this example, if we are given a height and an age of someone then we can work out the value of *k*.

$$k = \frac{height}{age}$$

Again, it's important to note that *k* is the gradient of the line we would get if we plotted a graph of height vs age.

Graphs and proportionality

Let's look at how all the work we have done with graphs relates to this!

With the equation $y = 3x$ when we plot it we see the graph on the right.

As the line is straight, we can say that y is **directly** proportional to x.

$$y \propto x$$

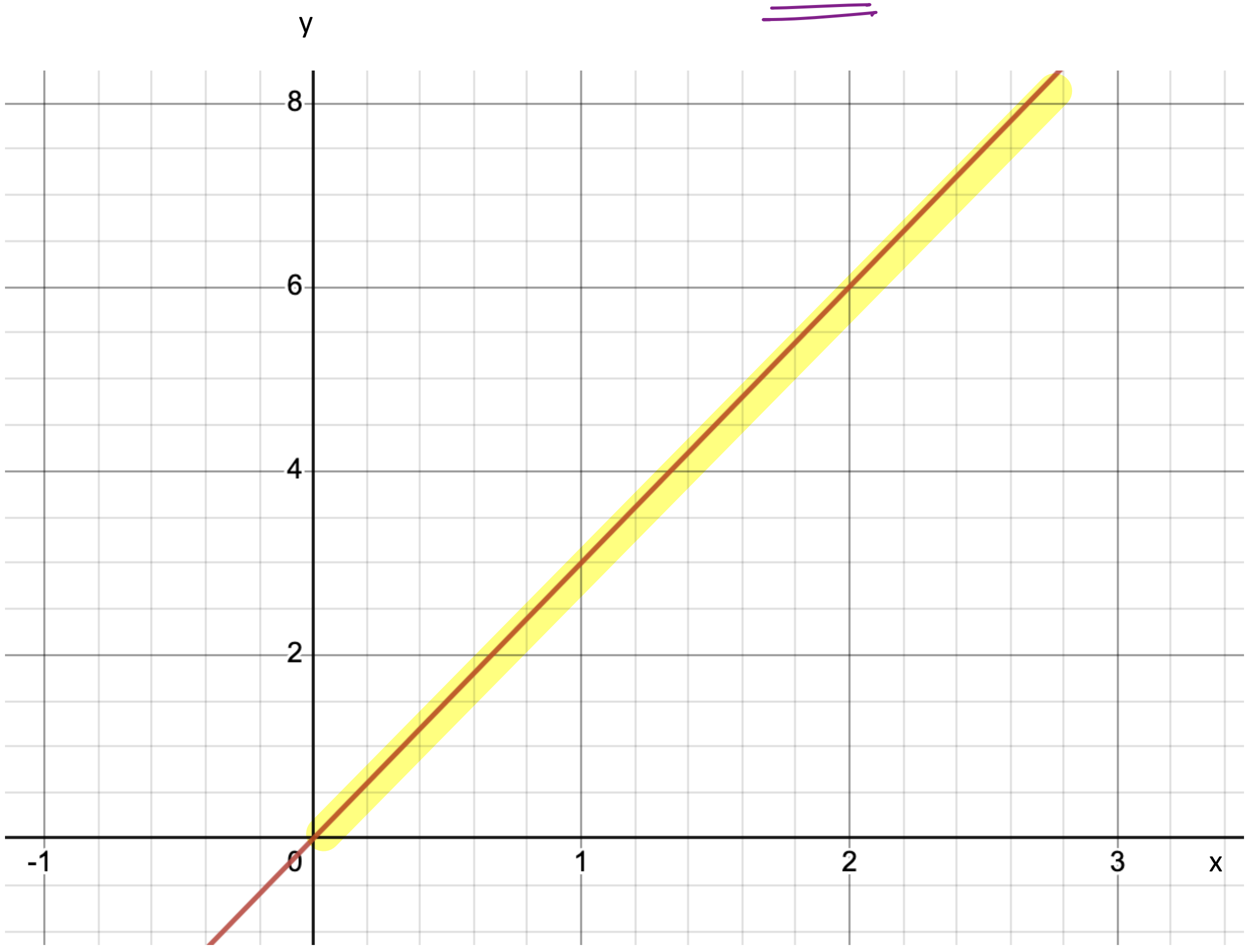
We know that the equation of the line is $y = 3x$

So, the constant of proportionality is 3 … which also happens to be the gradient!

$y = 3x$

$y = kx$

$k = 3$



Finding the constant of proportionality

We can use the ideas to help us find the value of *k* when given a table of values.

We are told that x and y are directly proportional of each other.

x	3	5	7	9
y	21	35	49	63

×7

Use the table to find *k* and the missing values

$y \propto x$

$y = k \times x$ F.

$21 = k \times 3$ S.

$k = 7$ S.

$\therefore y = 7 \times x$

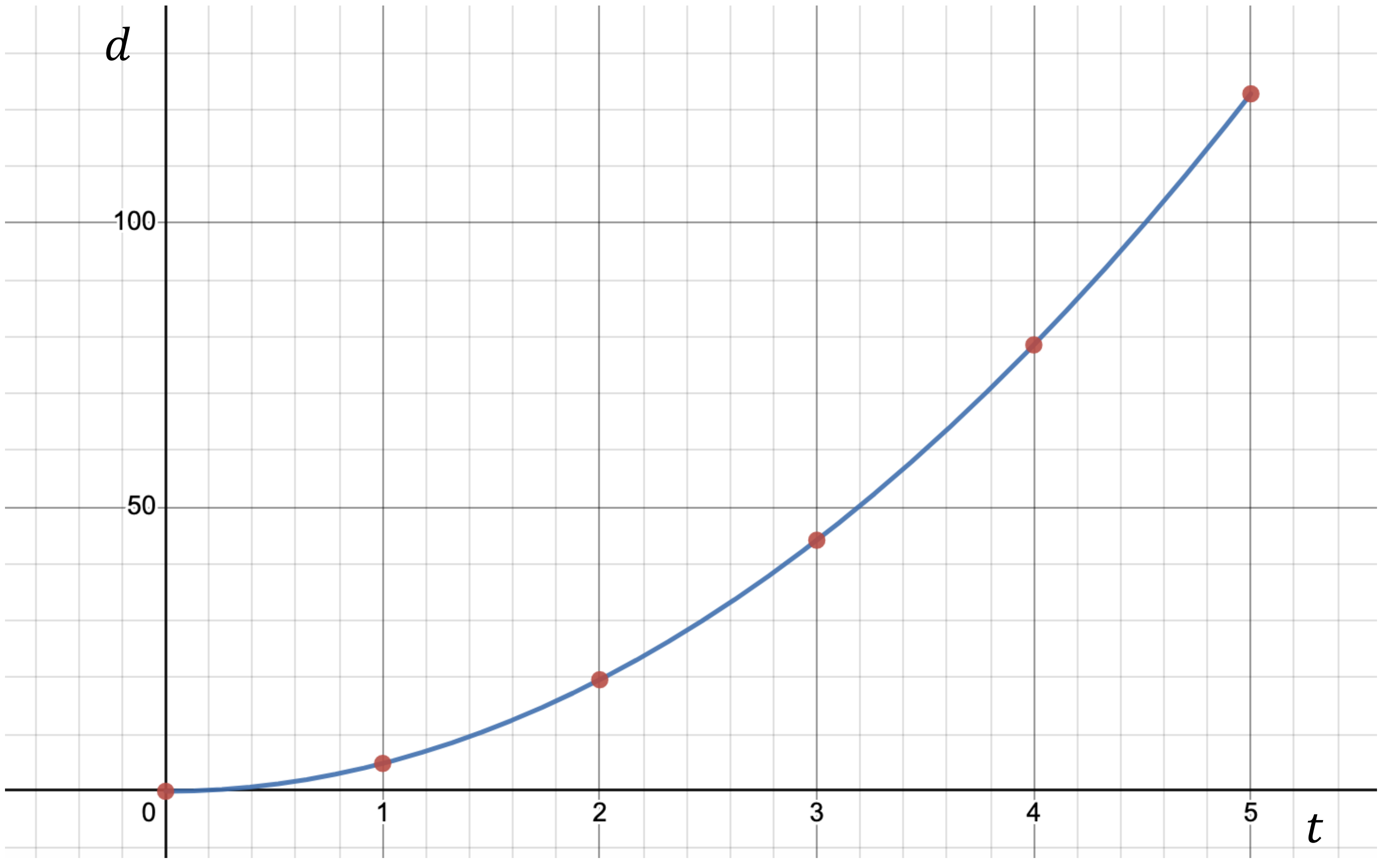
$63 = 7 \times x$

x

Not all graphs start as a straight line

Not all graphs start as a straight line, but we can perform some magic to make them straight.

If we look at the following data and the graph it provides, it's definitely not straight!



Time (t s)	0	1	2	3	4	5
Distance (d m)	0	4.91	19.64	44.19	78.56	122.75

Not all graphs start as a straight line

We can square the time's and we see the graph becomes that shown on the right.

This means, as it's a straight line, that:

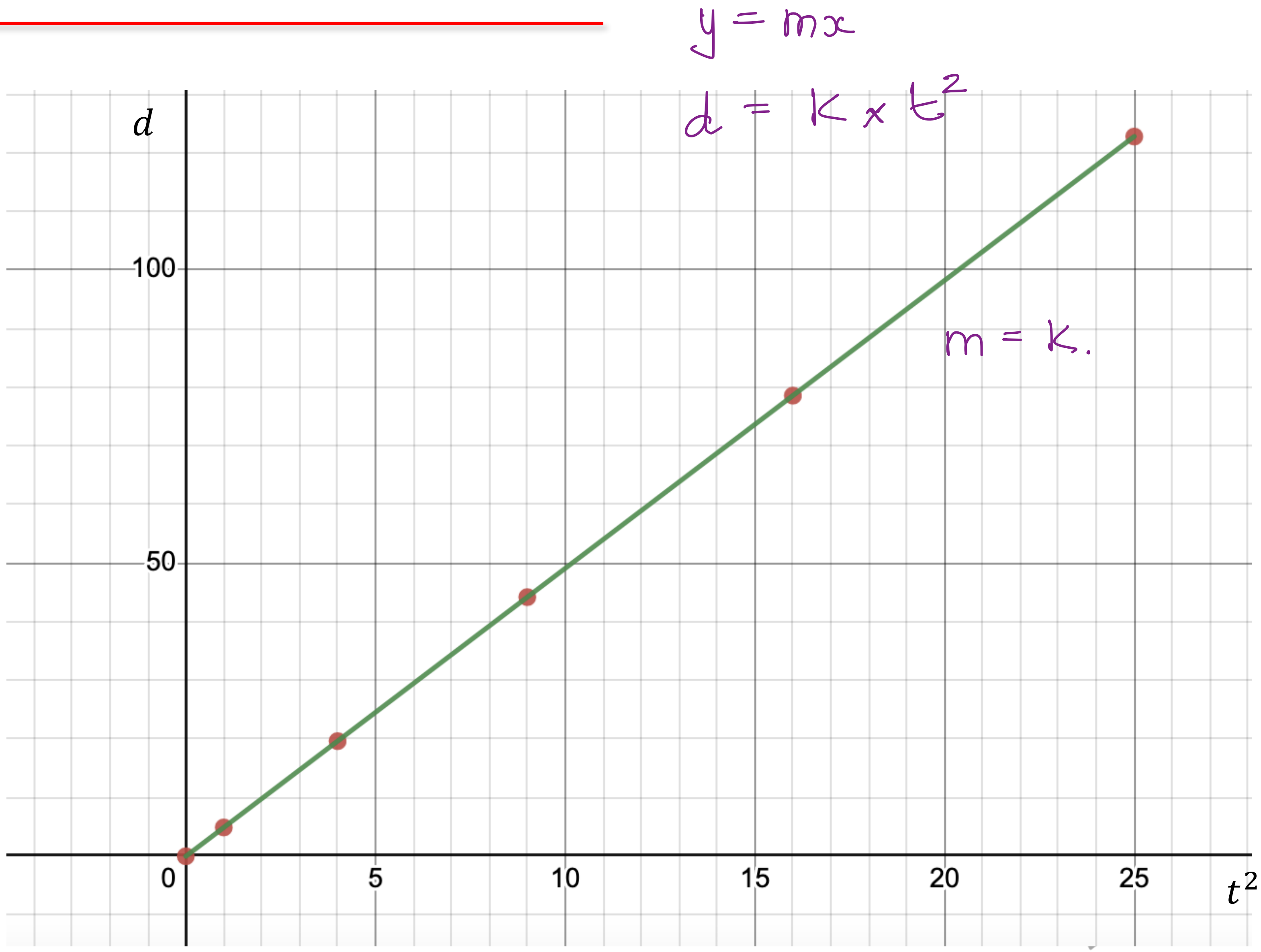
$$d \propto t^2$$

And we can use a couple of points to help us find the value of k when the equation becomes:

$$d = k \times t^2$$

$d \propto t$

$d \propto t^2$



They will normally give you what is proportional to what in the question!

Given that $y \propto x^2$, use the table of values to determine the constant of variation, k, and hence complete the table.

x	2	4	6	8
y	12	48	108	192

The good news is that you don't normally have to guess what is proportional to what as they give it to you in the question.

$y \propto x^2$

$y = k \times x^2$ F.

S.

$12 = k \times 4$

$k = 3$

$y = 3 \times x^2$

$y = 3 \times x^2$

$y = 3 \times 6^2$

$y = 108$

$y = 3 \times x^2$

$192 = 3 \times x^2$

$y = 3 \times 4^2$

$= 3 \times 16$

$= 48$

Example

Given that $y \propto x^2$, use the table of values to find the constant of variation and then complete the table.

x	2	4	6	12
y	8	32	72	288

$y \propto x^2$

$y = k \times x^2$

$32 = k \times 16$

$k = 2$

F

S

$y = 2 \times x^2$

$= 2 \times 2^2$

$= 2 \times 4$

$= 8$

$y = 2 \times x^2$

$288 = 2 \times x^2$

The good news is that you don't normally have to guess what is proportional to what as they give it to you in the question.

Example

They don't always have to give you a table! You only need two values to be able to find the value of *k* and then any missing values

In an electrical wire, the resistance (R ohms) varies directly as the length (L m) of the wire.

a If a 6 m wire has a resistance of 5 ohms, what is the resistance of a 4.5 m wire?

b How long is a wire for which the resistance is 3.8 ohms?

$R \propto L$

$R = k \times L$

$5 = k \times 6$

$k = 5/6$

$R = \frac{5}{6} \times L$

a. $R = \frac{5}{6} \times 4.5 = 3.75$ ohms

b. $R = \frac{5}{6} \times L$

$3.8 = \frac{5}{6} \times L$

$L = 4.56$ m

Example

A car is travelling at a constant speed. The distance travelled (d) varies directly as the time taken (t).

a If it takes 2 hours to travel 190 km, how far has the car travelled in 3 hours?

b How long does it take to travel 500 km? Give your answer in hours and minutes, rounded to the nearest whole minute.

They don't always have to give you a table! You only need two values to be able to find the value of *k* and then any missing values

$d \propto t$

$d = k \times t$ F

$190 = k \times 2$ S.

$k = 95$

$d = 95 \times t$

a. $d = 95 \times t$

$= 95 \times 3 = 285$ km.

b. $d = 95 \times t$

$500 = 95 \times t$

$t = 5.26315$ hrs

$t = 5$ hrs 16 mins

Work to complete

The work I am asking to be completed for this topic is shown below.

This is the minimum work which should be completed. The more questions which are answered the better your chance of success in exams. Questions towards the end of the exercises and in the Chapter Review are the best practice you can do.

Questions to complete:
Exercise 9A: 1, 2, 3, 4, 5, 6, 8, 10,

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