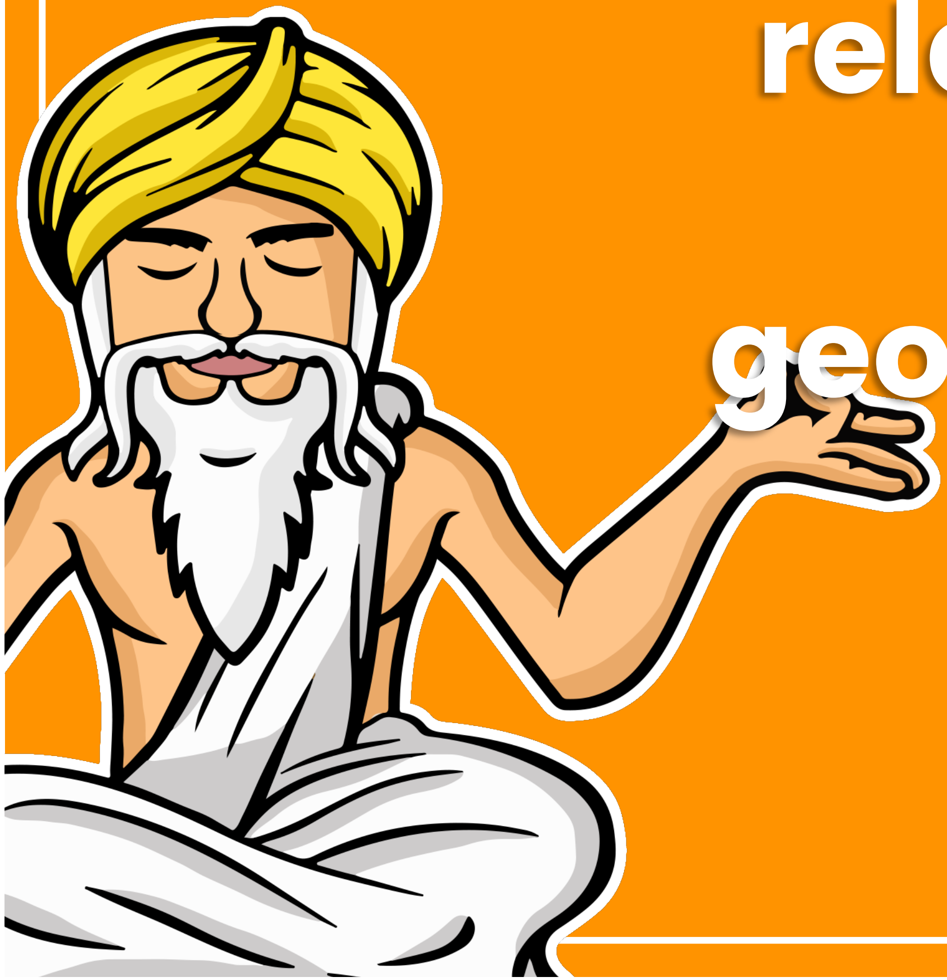
Using a recurrence relation to generate and analyse a geometric sequence



Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Know how to express a geometric sequence as a recurrence relationship.
- Know how to read an recurrence relationship
- Know how to generate a sequence using a recurrence relationship.

Recap of past learning

When we were looking at **arithmetic sequences** we found that we needed to be careful of the language we used!

There was a marked difference between a **rule** and a **recurrence relationship**. A rule allows us to to one specific term in a sequence. A recurrence relationship allows us to get a number in a sequence only when we have the previous number and the rule.

$t_{n+1} =$

$t_n =$

The recurrence relationship for an arithmetic sequence looked like:

$$t_0 = 100, t_{n+1} = t_n + 3$$

This lesson is going to look at how we can write geometric sequences in the same way!

$t_0 = 100, \; t_{n+1} = t_n \underline{\underline{+ 3}}$

The recurrence relationship for a geometric sequence

When we consider the sequence below, we see that the common ratio is 3. This means, to get from one term to the next we simply multiply the previous by 3.

$$2, 6, 18, \ldots$$

Knowing that a recurrence relationship must have t_1, t_{n+1} *and* t_n which stand for the first term, the next term and the current term, we can see that the following would be true:

$$t_1 = 2.\ t_{n+1} = 3 \times t_n$$

We can make this more general in the following way:

$$\boldsymbol{t_1 = a, t_{n+1} = r \times t_n}$$

$\rightarrow t_n$

t_n

$a = 2$

$r = 3$

$t_1 = 2, \quad t_{n+1} = t_n \times 3$

$t_1 = 2, \quad t_{n+1} = 3 \times t_n$

Example: Using a recurrence relationship to generate a sequence

a) Generate and graph the first five terms of the sequence defined by the recurrence relation:

$$t_1 = 5, t_{n+1} = 2t_n$$

b) Use a rule to calculate the value of the 10th term in the sequence.

term	num
1	5
2	10
3	20
4	40
5	80

$t_{n+1} = 2t_n$

$t_n = a \times r^{n-1}$

$t_{10} = 5 \times \boxed{2^9}$

$= \underline{\underline{2560}}$

$5 \cancel{\times 2}^9$

Example: Application

The volume of cube 1 is 8 cm^3.

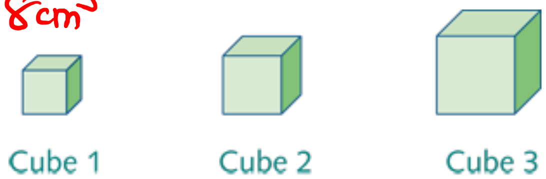
The volume of each successive cube is 1.5 times the volume of the previous cube.
Let V_n be the volume (in cm^3) of the nth cube in this sequence of cubes.

A recurrence relation that can be used to generate the volumes of this sequence of cubes is:

$$V_1 = 8\ cm^3, \qquad V_{n+1} = 1.5V_n$$

a) Use the recurrence relation to generate the volumes of the first four cubes in this sequence and use these volumes to construct a table showing the cube number (n) and its volume (V_n).
b) Use the table to plot the volume of the cube against cube number and comment on the form of the graph.
c) Use the rule for the nth term for this sequence to predict the volume of the 10th cube in this sequence.

$8cm^3$ $\times 1.5$ $\times 1.5$



$a = 8$

$r = 1.5$

a)

n	1	2	3	4
V_n	8	12	18	27

b) non-linear and increasing

c) $t_n = a \times r^{n-1}$

$t_{10} = 8 \times (1.5)^9$

$= 307.55\ cm^3$