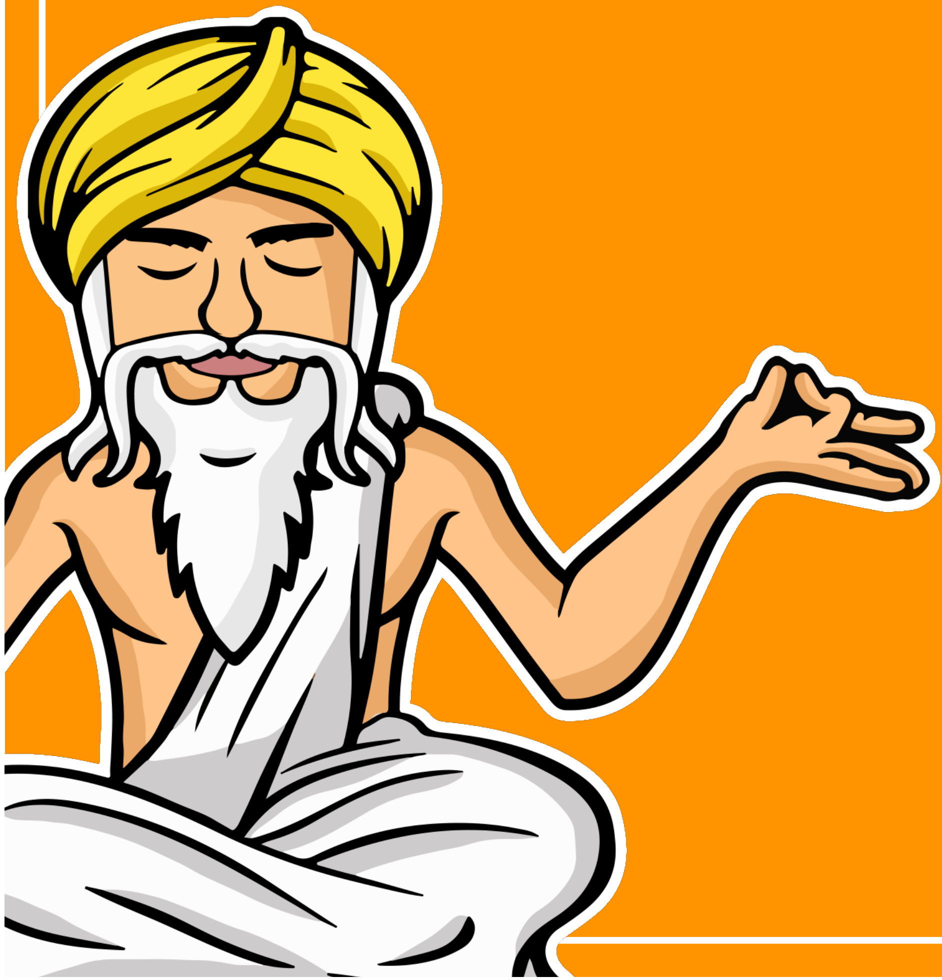

Geometric sequence applications

Year 11
General Mathematics

www.maffsguru.com

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Know how to find the nth term of a geometric sequence
- Know how to use the formula to find the nth term of a geometric sequence
- Understand how to find the common ratio when given a percentage change
- Be able to apply the knowledge to real world situations

Recap of past learning

We have now started to look at the really interesting stuff! We have, in the previous videos, understood what an **arithmetic sequence is** and now we know what a **geometric sequence** is.

We found that there was a rule we could use for arithmetic sequences which would allow us to find the nth term of any sequence:

$$t_n = a + (n-1) \times d$$

Wouldn't it be nice to be able to use something like this for geometric sequences?

Well, as it turns out, we can!

$$t_n = a + (n-1) . d$$

Finding the nth term of a geometric sequence

Let's consider the following sequence and see if we can build up a formula for the nth term:

$2, 6, 18, 54, 162, 486$

It might be easier if we can express it in a table (like we did for arithmetic sequences):

term	t_1	t_2	t_3	t_4
number	2	6	18	162

$r = 3$

$t_1 = 2 \qquad \underline{\underline{a = 2}}$

$t_2 = 2 \times 3 = 6$

$t_3 = 2 \times 3 \times 3 = 2 \times 3^2$

$t_4 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

$t_5 = 2 \times 3^4$

$t_6 = 2 \times 3^5$

$t_n = a \times r^{n-1}$

Finding the nth term of a geometric sequence

So, the formula we can use is:

$$t_n = a \times r^{n-1}$$

rule

Where:

a is the first term,
r is the common ratio, and
n is the term number we are looking for.

Simples!

Example: Finding the nth term of a geometric sequence

Find t_{15}, the 15th term in the geometric sequence: $3, 6, 12, 24, \ldots$

$t_n = a \times r^{n-1}$

$t_{15} = 3 \times 2^{14} = \underline{\underline{49\,152}}$

$\frac{6}{3} = 2$

$n = 15$

$a = 3$

$r = 2$

$\frac{12}{6} = 2$

Turning percentage change into a common ratio

In real world applications we are told that prices are going to go up or down in terms of a percentage.

For example, the cost of apples is going to rise by 10%.

In the first semester, this course showed us that we could turn this percentage into a decimal multiplier.

All we need to know is **percentage** and whether the price is going **up** or **down**.

$\uparrow 10\%$

$100\% + 10\% = 110\%$

$\downarrow \div 100$

1.1

$50 \times 1.1 = 55$

5

Turning percentage change into a common ratio

A 10% increase can be expressed as a percentage multiplier:

20% decrease

$100\% - 20\% = 80\%$

$\downarrow \div 100$

0.8

$\$60 \times 0.8 = \$48$

Turning percentage change into a common ratio

A 20% decrease can be expressed as a percentage multiplier:

Inc 15%

$100 + 15 = 115\%$

$\downarrow \div 100$

$\underline{\underline{1.15}}$

dec 13%

$100 - 13 = 87\%$

$\downarrow$

$\underline{\underline{0.87}}$

Rule for turning percentage change into a common ratio

When there has been a P% **increase** we can find the common ratio as:

$$r = 1 + \frac{P}{100}$$

10%

$$r = 1 + \frac{10}{100} = 1.1$$

When there has been a P% **decrease** we can find the common ratio as:

$$r = 1 - \frac{P}{100}$$

30%

$$r = 1 - \frac{30}{100} = 0.7$$

Notice that the formulas are the same except for the plus and minus sign.

This corresponds to whether you are increasing (plus) or decreasing (minus).

Example: Calculating the common ratio for a given percentage change

State, correct to two decimal places, the first four terms in each of these geometric sequences for the changes given.

a) Starts at 200 and each new term is 4% less than the previous term.
b) Starts at 500 and each new term is 12% more than the previous term.

$\downarrow 4\%$

$100 - 4$

$= 96\%$

$\downarrow$

0.96

200

192

184.32

176.95

Example: Calculating the common ratio for a given percentage change

State, correct to two decimal places, the first four terms in each of these geometric sequences for the changes given.

a) Starts at 200 and each new term is 4% less than the previous term.
b) Starts at 500 and each new term is 12% more than the previous term.

$100 + 12 = 112\%$

$\downarrow$

1.12

500

560

627.2

702.46

Example: Application

As a park ranger, Megan has been working on a project to increase the number of rare native orchids in Wilsons Promontory National Park.

At the start of the project, a survey found 200 of the orchids in the park. It is assumed from similar projects that the number of orchids will increase by about 18% each year.

a) State the first term a, and the common ratio r, for the geometric sequence for the number of orchids each year.
b) Find a rule for the number of orchids at the start of the nth year.
c) How many orchids are predicted in 10years time?

$100 + 18$

118%

$\downarrow$

1.18

$\boxed{200}$

a) $a = 200$

$r = 1.18$

b) $t_n = a \times r^{n-1}$

$t_n = 200 \times (1.18)^{n-1}$

c) $t_{10} = 200 \times (1.18)^9 = \underline{\underline{888}}$