Geometric sequences



Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Understand what a geometric sequence is and be able to identify it
- Understand the difference between an arithmetic and geometric sequence
- Know what the common ratio is and how to find it
- Know how to use the CAS to find terms of a geometric sequence
- Know what a graph of a geometric sequence looks like

Recap of past learning

In the previous lessons we have looked at what an **arithmetic sequence** is. This is where we know that the difference between one term and the next is a result of **adding** or **subtracting** the same number (called the common difference).

An arithmetic sequence is **linear** as can be seen when we plot the points on a graph.

But not all sequences are linear! When we put money into a bank account we accrue interest. This interest then accrues interest. The more we have the more it grows. It does not grow in a linear fashion! So, we need something to describe this.

Welcome to the lesson on geometric sequences.

2 4 6 8 10

10 7 4 1 -2

What is a geometric sequence?

In short, this is when we multiply one term to get to the next term.

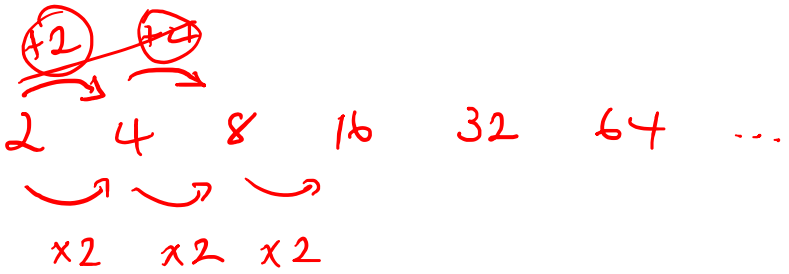
To go from term to term we **must** multiply by the same number.

Note: **We never divide as we can turn a division into a multiplier.**

An example of a geometric sequence is shown below:

$$2, 4, 8, 16, 32, 64, 128, 256, 1024 \ \ldots$$

We can see that there is **no common difference** but that to go from one term to the next we seem to be **doubling it**. This is a perfect example of a geometric sequence.



The common ratio

When we were looking at Arithmetic sequences we had a fancy term to describe the difference. It was called the **common difference**.

When we are using a multiplier … we call it the **common ratio** and, in mathematics, this is given the letter 'r'.

So, for the previous example:

$$2, 4, 8, 16, 32, 64, 128, 256, 1024 \ \ldots$$

$r = 2$

We would say the common ratio, r, is 2.

$d \neq$

$r = 2$

We never divide!

Common ratios can be fractional too.

When we divide a number by 2, we are really multiplying by $\frac{1}{2}$.

So, the following sequence would have $r = \frac{1}{2}$.

$$100, 50, 25, 12.5, 6.25, \ldots$$

There is nothing to say that we cannot have decimal numbers in a sequence by the way!

100 ↓ ÷2
50
25

$10 = \frac{10}{1}$

$2 = \frac{2}{1}$

$3 = \frac{3}{1}$

÷2

× $\frac{1}{2}$

$r = \frac{1}{2}$

Finding the common ratio

There has to be a nice way to find the common ratio!!!

When we did Arithmetic sequences, we could find the common difference by using the following formula:

$$common\ difference = t_2 - t_1$$

Where the terms were denoted as t_1, t_2, t_3, t_4, etc

For **geometric sequences** we use the same idea to come up with the following:

$$common\ ratio = \frac{t_2}{t_1}$$

Remember: To need to check a few of the terms to make sure that the common difference is the same!!! For example, the following is **not** a geometric sequence:

$$1, 4, 9, 16, 25, 36, 49, \ldots$$

$4 \div 1 = 4$ $\quad 9 \div 4 \neq 4$

Examples: Finding the common ratio

Find the common ratio in each of the following geometric sequences.

- 3,12,48,192, ...
- 16,8,4,2, ...

$r = 4$

$r = \frac{1}{2}$

3 12 48 192

$12 \div 3 = 4$

$48 \div 12 = 4$

16 8 4 2

$\frac{8}{16} = \frac{1}{2}$ $\quad \frac{4}{8} = \frac{1}{2}$

Examples: Identifying a geometric sequence

Which of the following is a geometric sequence?

- 2,10,50,250, ... ✓
- 3,6,18,36, ... ✗

$2 \quad 10 \quad 50 \quad 250$

$\frac{10}{2} = 5 \qquad \frac{50}{10} = 5$

$3 \quad 6 \quad 18 \quad 36$

$\frac{6}{3} = 2 \qquad \frac{18}{6} = 3$

Using the CAS to find the terms of a geometric sequence

This is a CAS enabled course so there must be a way to find the terms of a geometric sequence! And, as it turns out, there is. And we use the same idea as we did for arithmetic sequences.

Example: *Generate the first six terms of the geometric sequence*: $1,3,9,27,\ldots$

1 3 9 27 81 243

$a = 1$

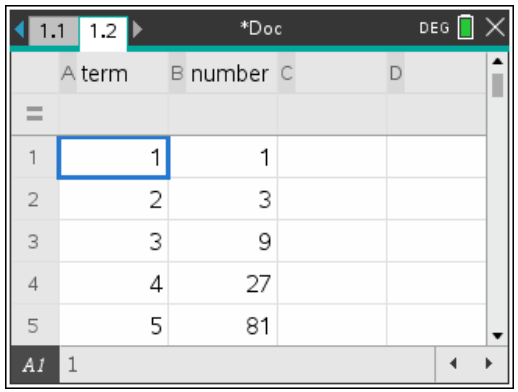
$r = \frac{3}{1} = 3$

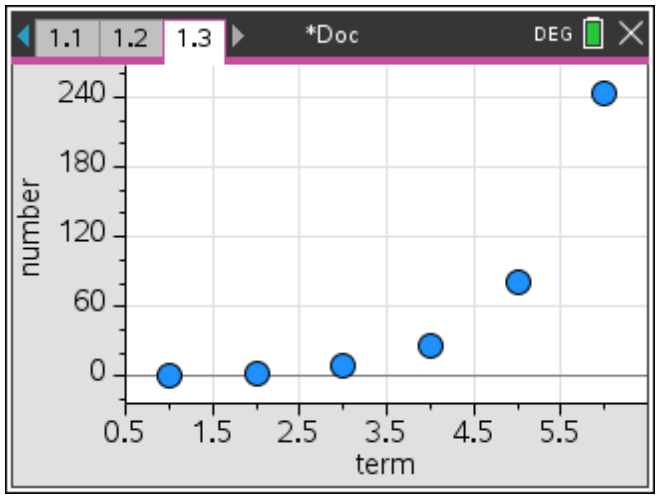
What does the graph of a geometric sequences look like?

Funny you should ask!

If we take the previous example and put it into a graph, we can see that, rather than being linear, the graph appears to curve upwards.

It's important to note that the curve grows at an ever increasing rate!





Example: Graphing a Geometric sequence

Consider the geometric sequence: 2,6,18, ...

a) Find the next term.
b) Show the positions and values of the first four terms in a table.
c) Use the table to plot the graph.
d) Describe the graph.

2 6 18 54

$a = 2$

$r = 3$

$\frac{6}{2} = 3$

t_1	t_2	t_3	t_4
2	6	18	54

non linear ascending

Example: Graphing a Geometric sequence

Consider the geometric sequence: 32,16,8, ...

a) Find the next term.
b) Show the positions and values of the first four terms in a table.
c) Use the table to plot the graph.
d) Describe the graph.

32 16 8 4

$\frac{16}{32} = \frac{1}{2}$

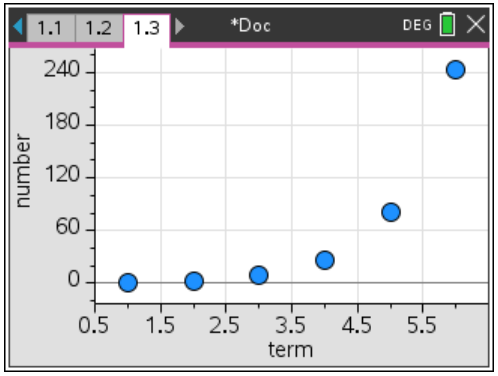
$r = \frac{1}{2}$

t_1	t_2	t_3	t_4
32	16	8	4

non-linear decending

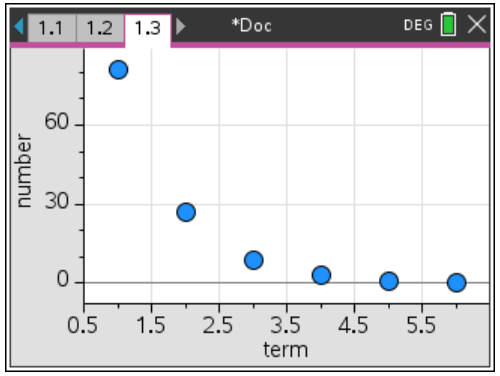
Things to note!!

When the common ratio is greater than 1, we see the curve will be increasing.



Common ratio is 3

When the common ratio is less than 1, we see the curve will be decreasing.



Common ratio is $\frac{1}{3}$