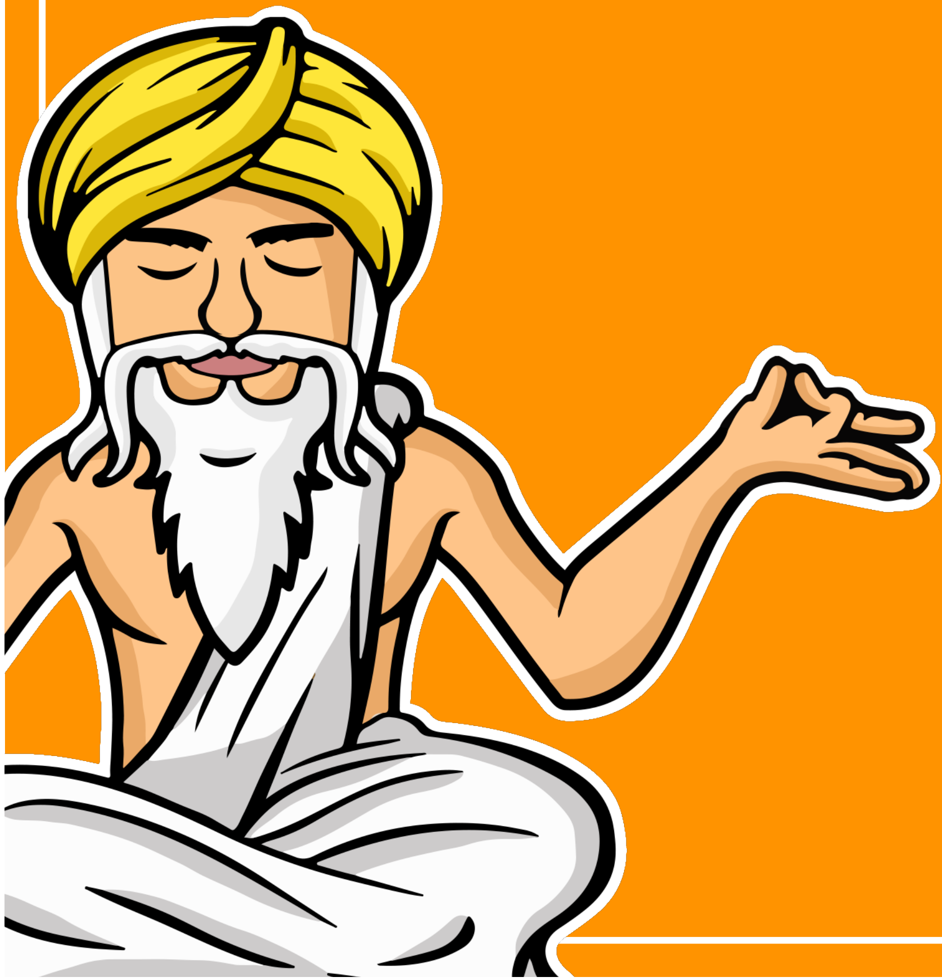
Arithmetic sequence applications



Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Understand how to apply arithmetic sequences.
- Know how to find the nth term of an arithmetic sequence.

Recap of past learning

We have been looking at what a sequence is and how it might be created. Firstly we looked at how to create a rondom sequence of numbers. Whilst useful, it doesn't allow us to predict numbers or to continue the sequence.

We then looked at term-to-term sequences which, in Year 12, are called recurrence relationships. This is when we have a "rule" or "pattern" which allows us to get from one number to the next.

There are lots of sequences we can make. Two of the main types we will be looking at are **arithmetic** and **geometric** sequences.

Arithmetic sequences are those when the difference between each term is the same. This differences is called the **common difference**.

Now, we can look at how we might extend our understanding of sequences and, in particular, arithmetic sequences.

+2

2, 4, 6, 8, 10, 12, 14 ....

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

How to find a term which isn't close to the ones we have been given

We might have been given the number sequence below:

2, 4, 6, 8, 10, …

Being asked to find the next three terms in the sequence isn't particularly difficult.

Because we know the common difference (+2), we can simply add two on each time. We can either do this in our heads or using the CAS.

But what if we wanted to find the 50th number in the sequence? Or the 100th?

There **must** be a way we can do this.

$d = 2$

$d = -3$

10 12 14 16

100 102 104 106

10 7 4 1 −2 …

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How to find a term which isn't close to the ones we have been given

Turning the list into a horizontal table can sometimes make things a bit easier!

Let's look at how we go from each number to the next.

Term	1	2	3	4	5
Term	t_1	t_2	t_3	t_4	t_5
Number	2	4	6	8	10

$t_1 = 2$

$t_2 = 2 \boxed{+ 2} = 2 + 1 \times 2$

$t_3 = 2 \boxed{+ 2 + 2} = 2 + 2 \times 2$

$t_4 = 2 \boxed{+ 2 + 2 + 2} = 2 + 3 \times 2$

$t_5 = 2 + 4 \times 2$

$t_n = \boxed{2} + (n-1) \boxed{\times 2}$

d = common diff

nth term

a = first term

$t_1 = 2$

$a = 2$

$t_n = a + (n-1) \times d$

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Rule for finding the nth term of an arithmetic sequence

If we have an arithmetic sequence, the rule for finding the nth term is:

$$t_n = a + (n-1)\,d$$

Where a is the first term, n is the term we are looking for and d is the common difference.

$a = 3$

$d = 4$

$t_n = a + (n-1) \cdot d$

$3, 7, 11, 15 \ldots$

t_{20}

$t_n = a + (n-1) \cdot d$

$t_{20} = 3 + (20-1) \cdot 4$

$= \underline{\underline{79}}$

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

Example: Finding the nth term of an arithmetic sequence.

Find t_{30}, the 30th term in the arithmetic sequence:

21, 18, 15, 12, …

t_{30}

$d = 18 - 21$

$= -3$

$t_n = a + (n-1) \cdot d$

$t_{30} = 21 + (30-1)(-3)$

$= -66$

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

Example: Finding the nth term of an arithmetic sequence.

Find t_{35} in the following arithmetic sequence:

18, 21, 24, 27, …

t_{35}

$d = 21 - 18$

$= \underline{\underline{3}}$

$t_n = a + (n-1).d$

$t_{35} = 18 + (35-1).3$

$= \underline{\underline{120}}$

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

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Example: Finding the nth term of an arithmetic sequence.

Find the 40th term in an arithmetic sequence that starts at 11 and has a common difference of 8.

$$t_n = a + (n-1).d \qquad a = 11$$

$$t_{40} = 11 + (40-1).8 \qquad d = 8$$

$$= 323$$

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

Example: Application of an arithmetic sequence.

The hire of a car costs \$180 for the first day and \$150 for each extra day.

a) How much would it cost to hire the car for 7 days?
b) Find a rule for the cost of hiring the car for n days.

$a = 180$

$d = 150$

180
$\boxed{+150}$
+150
+150

a) $t_n = a + (n-1).d$

$t_7 = 180 + (7-1).150$

$= \$1080$

b) $t_n = a + (n-1).d$

$t_n = 180 + (n-1).150$

$= 180 + 150n - 150$

$= 150n + 30$

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook