Arithmetic sequences

Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Understand what an arithmetic sequence is
- Know how to identify an arithmetic sequence
- Know what it means to be a:
    - Common difference
- Know how to use the CAS to generate arithmetic sequences
- Know what the graph of an arithmetic sequence would look like

Recap of past learning

In the last lesson we looked at what it meant to be a sequence. We know there are random sequences where there is no pattern between the numbers in a sequence. We are more interested in looking at those sequences where there are patterns.

In this lesson we are going to look at those sequences where the difference between each successive number is the same.

Common Difference: The same difference between terms

In the previous video we looked at a number of sequences and wondered if we could find a pattern which connected the numbers.

- 2, 8, 14, 20, … ← +6 $d = 6$
- ~~5, 15, 45, 135, …~~
- 7, 4, 1, −2, … ← −3 $d = -3$
- ~~1, 4, 9, 16,~~ …
- ~~1, 1, 2, 3, 5,~~ …

2 4 6 8 10

3 6 9 12

"d"

In some of them we found that we could simply add or take the same number away each time we went from one term to the other.

Others we needed to know the position of the number in the line to help us find each number in the sequence.

This lesson we're going to deal with the sequences where the difference between each number is the same. This is going to be called the **common difference**.

Examples have been extracted, with permission, from the Cambridge General Mathematics Units 1 and 2 Textbook

Common Difference: Finding the common difference

If ever we want to find the common difference all we need to do is take two consecutive numbers and subtract them!

So, finding the common difference of the following sequences should be simple:

- 2, 5, 8 …
- 25, 23, 21 …

My advice is to always do $t_2 - t_1$ and don't get tricked with the negative numbers!

$t_2 - t_1$

$2, 5, 8, \ldots$

$8 - 5 = \underline{\underline{3}}$

$5 - 2 = \underline{\underline{3}}$

$d = 3$

$25, 23, 21$

$23 - 25 = -2$

$d = \underline{\underline{-2}}$

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Arithmetic sequences

When the common difference is the same, we call the sequence an **arithmetic sequence**.

Remember we can add or subtract numbers!!

So, which of the following are arithmetic sequences?

- 21, 28, 35, 42 ✓
- 2, 6, 18, 54 ✗

+

−

21 28 35 42

7 7 7

d = 7

2, 6, 18, 54

$\boxed{+4 \quad 12}$

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Arithmetic sequences: Using a CAS to find them

We can use the CAS to help us find sequences of numbers by using the fact that it stores our "previous answer or term" in the ANS memory.

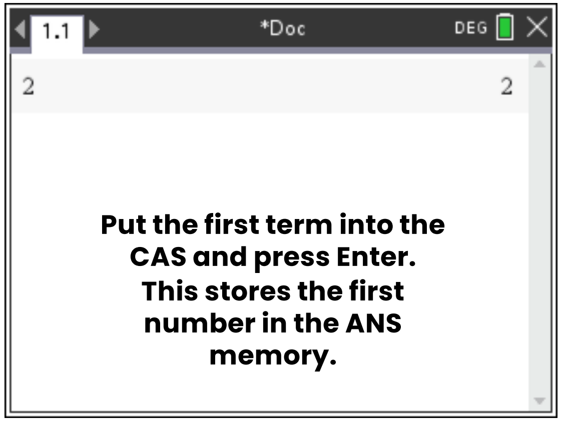
Shown below is an example of how to create a "add four" sequence when we start with 5.

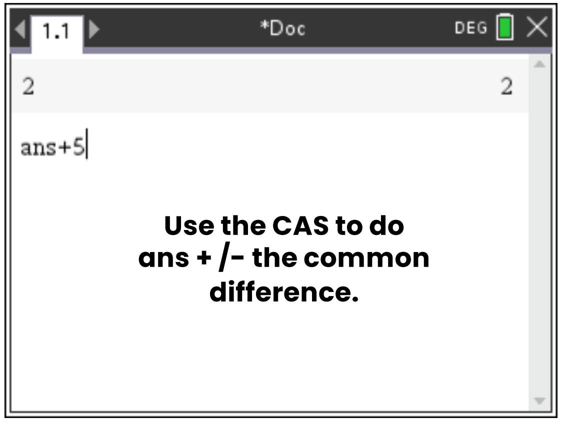
Note: It is important to know the number you are starting the sequence from. They will normally always give this to you in the question.

So, for example, if I wanted to create the number sequence 2, 7, 12 …

$\boxed{2}$

$\boxed{+5}$





Each time you press Enter your CAS will give you the next number in the sequence.
MAGIC!

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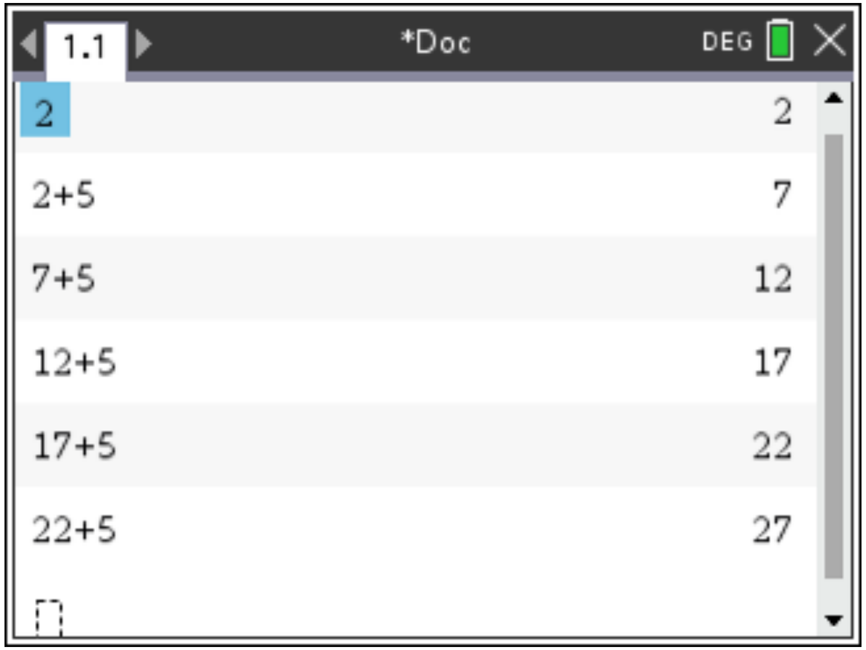

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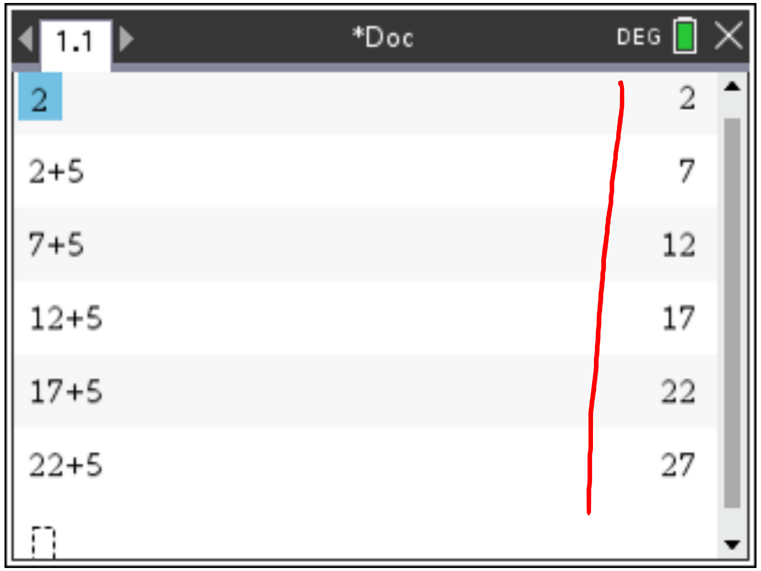
What does an Arithmetic sequence look like?

We know that we can use the CAS to show us relationships between things.

If we remember, a sequence is a list of numbers which have a position in a queue.

So, the numbers we got in the previous example, can be expressed in a table too.



2 t_1
7 t_2
12 t_3
17 t_4
22 ⋮
27

Term	1	2	3	4	5
number	2	7	12	17	22

This can now be put into a list and spreadsheet on the CAS and graphed.

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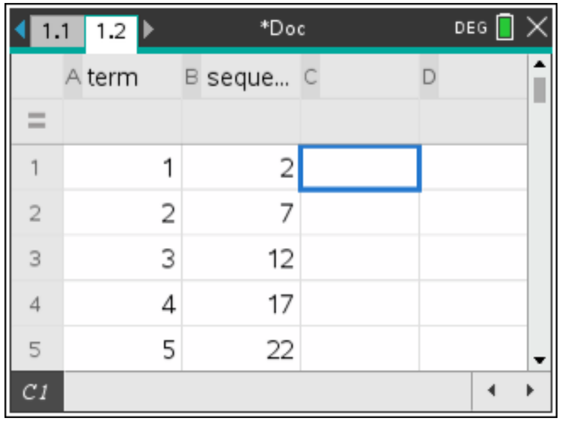
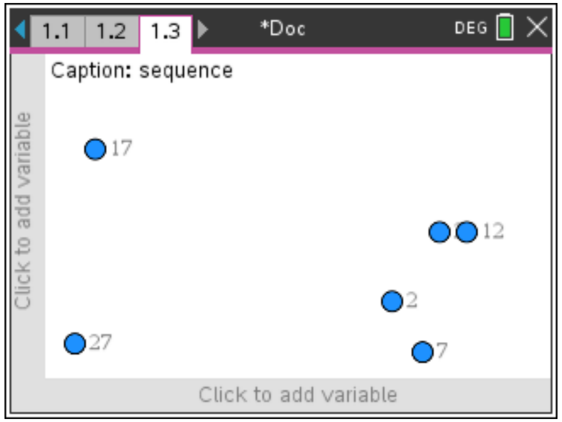
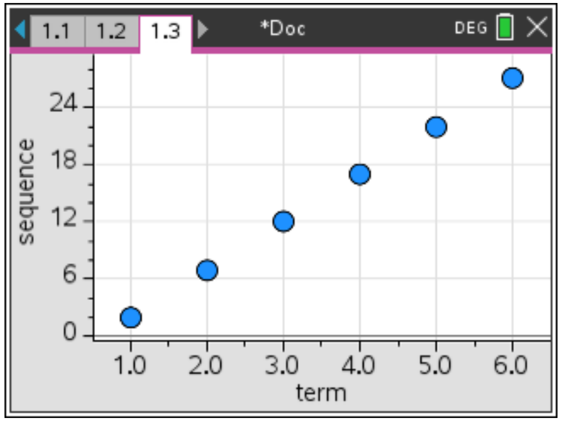
What does an Arithmetic sequence look like?







We can see, from the graph, that the arithmetic sequence is a straight line.

This makes sense as the common difference is the same.

All, arithmetic sequences, should give us a straight line graph which might slope up or down depending on the common difference.

Term	1	2	3	4	5
number	2	7	12	17	22

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Example: Graphing a decreasing arithmetic sequence

A decreasing arithmetic sequence would mean that the common difference would be negative.

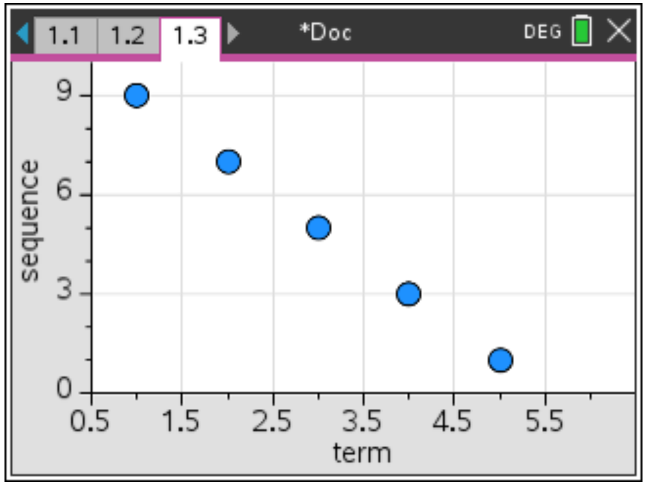
Let's see what the graph of the sequence 9, 7, 5, … looks like.

9, 7, 5, ….

The common difference is −2 so we can draw a table of results:

Term	1	2	3	4	5
number	9	7	5	3	1

And then use the CAS to show how this would look.



Note that the gradient of the slope is negative.

This is because the common difference is negative.

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