

Interpreting and predicting from a linear model

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

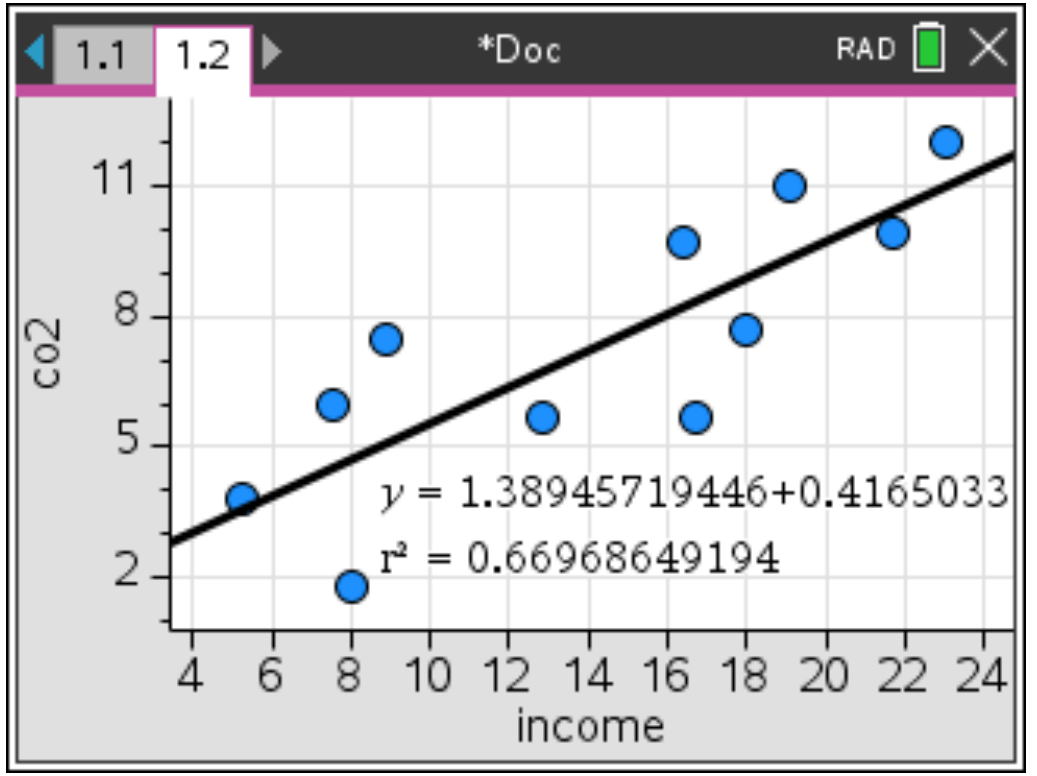
- Be able to understand what an equation of a leaast squares line helps us do
- Use the equation to predict values for both the EV and the RV
- Understand what it means by ‘interpolation’ and ‘extrapolation’



Recap of past learning

In the last lesson we looked at how we can use the CAS to enter a number of data values and plot them on a scatterplot. We then ask the CAS to find the equations of the least squares line (and r-squared).

The reason we do this is to allow us the **predict values for both the EV and RV which we might not have actual data values for**.





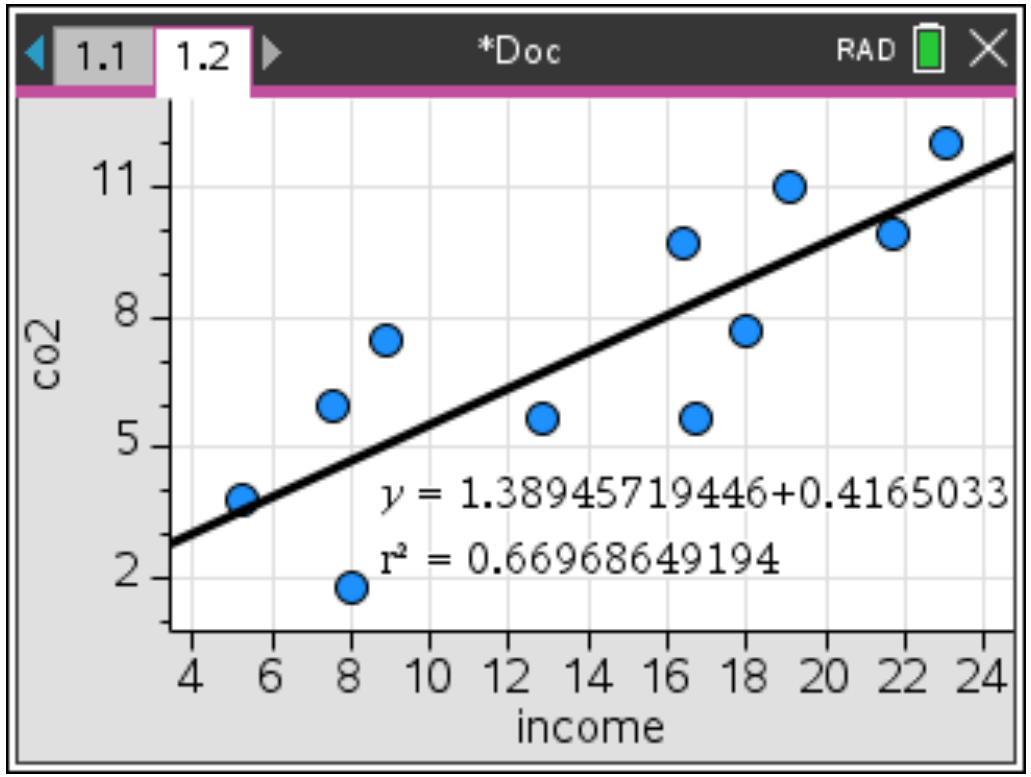
Understanding the gradient and intercept

In all the years we have taught you about the **gradients** and **intercept** have we really taught you what they are?

General Maths is about you **understanding** the content and not just regurgitating it. You need to understand exactly what it means by the gradient and intercept.

Good news is … the General Maths course helps us with boilerplate sentences where we simply change some numbers.

Let's use the same example from previous lessons and slides.



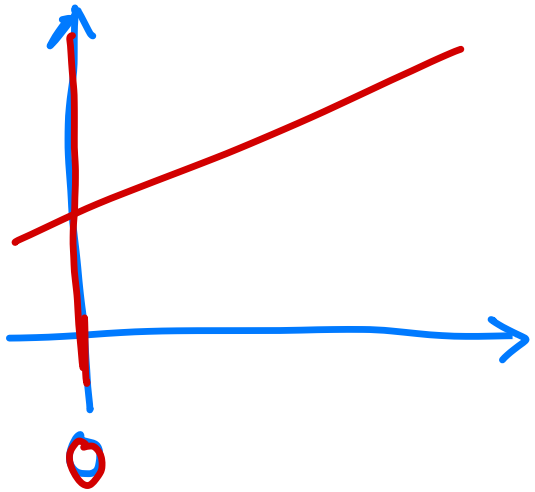


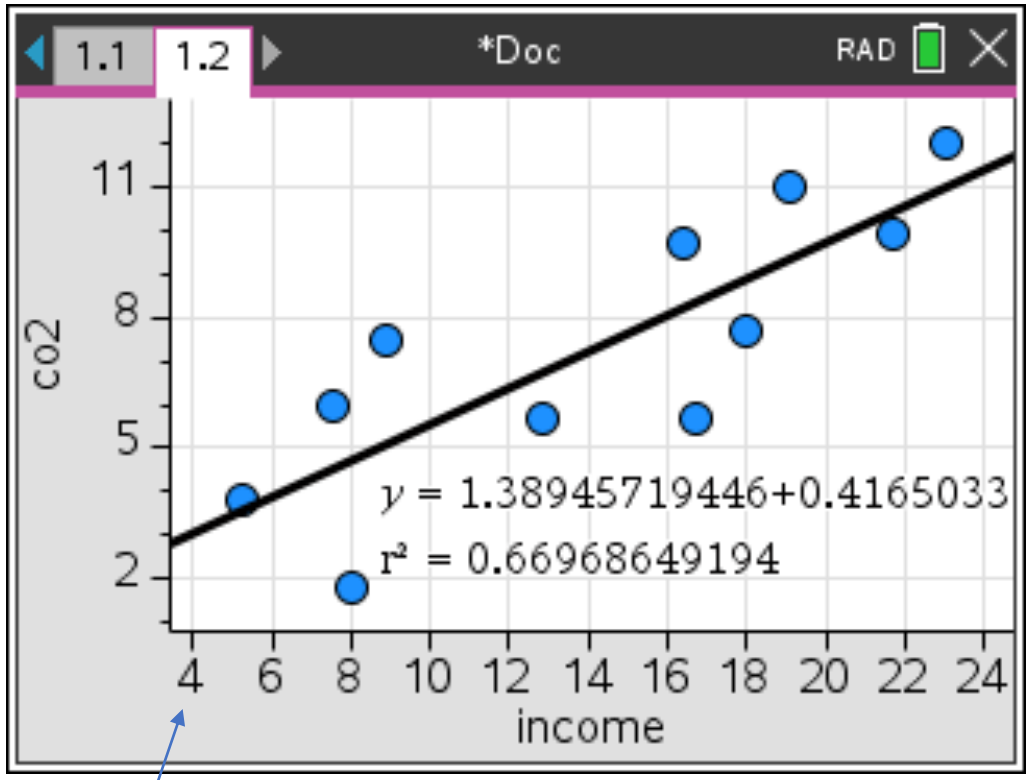
The intercept

Beware! Your CAS might lie to you!!!

What is the intercept from the graph shown on the right?

You **might** think it's about 3 ... as this seems to be where it crossed the vertical axis. But it's not!





The intercept is where the lines crosses when the x-axis is zero



Interpreting the intercept

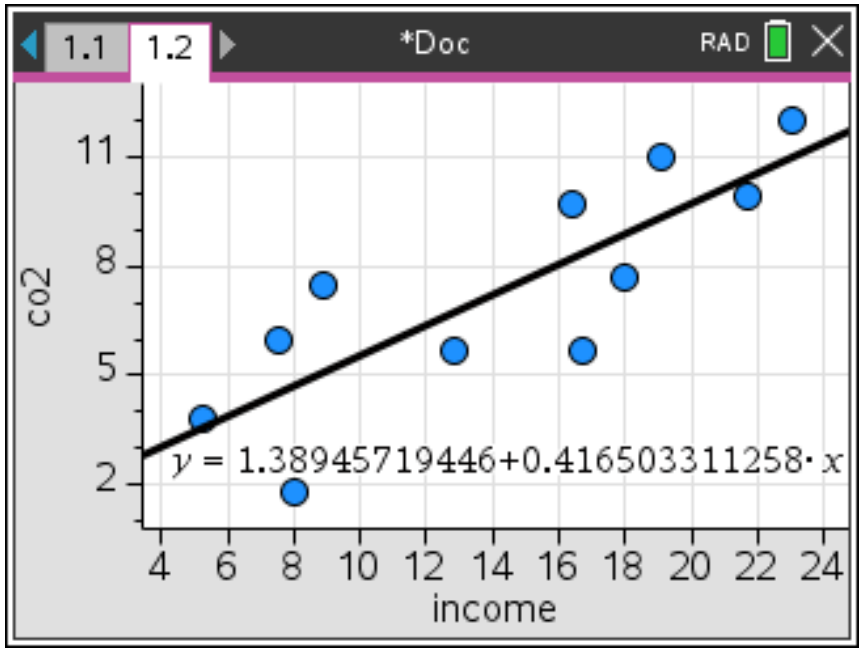
You will be expected to write a sentence **interpreting** the intercept. It is the value of the RV when the EV is zero. So, this is effectively what we put.

As we have taken a lot of values and **averaged them out** we must start the sentence with "on average".

Using the formula from the graph (rounded to 3 s.f.):

$$co2 = 1.39 + 0.417 \times income$$

$y = a + b \cdot x$



The intercept tells us, **on average**, that for **zero dollars in income**, there would be **1.39 tonnes of C02** released into the atmosphere.



What is a gradient?

The gradient is awesome!

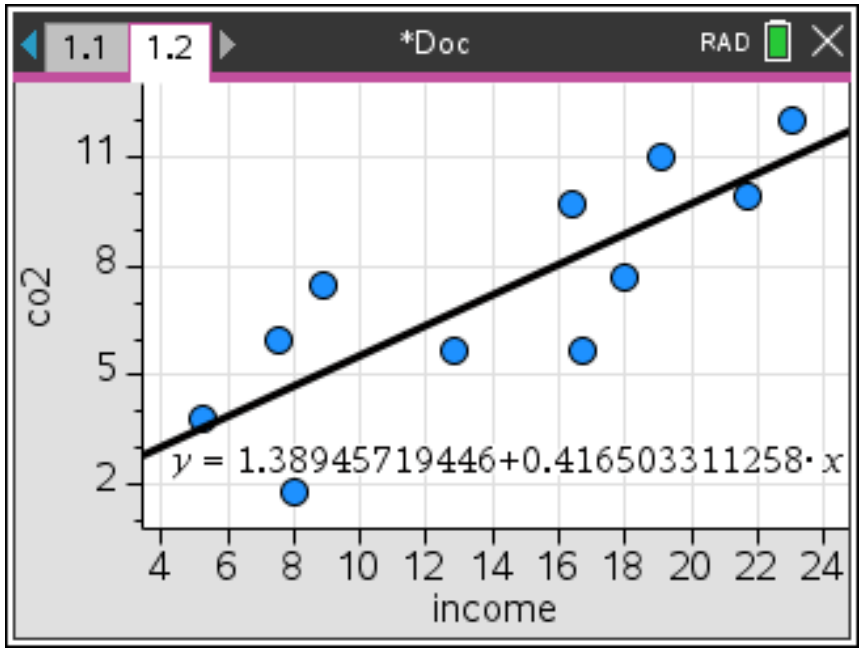
It tells us by how much the **RV** is **increasing** or **decreasing** for **every one unit increase** in the **EV**.

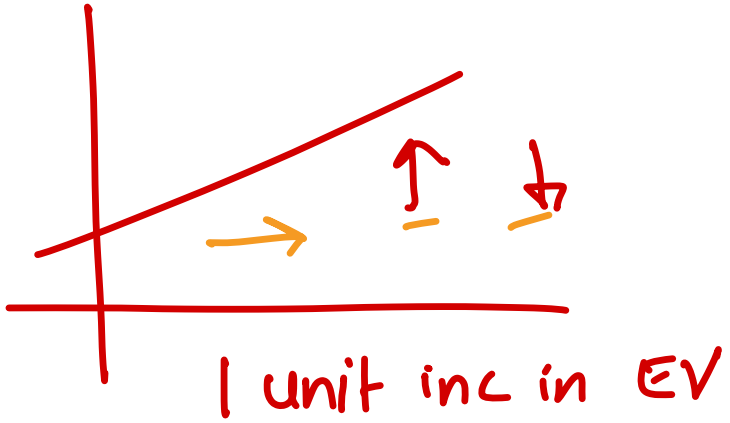
$$co2 = 1.39 + 0.417 \times income$$

So, for the above example, the gradient is 0.417.

We **always** relate this to one unit increase in the EV.

The positive (or negative) sign tells us whether the gradient is **increasing** or **decreasing.**





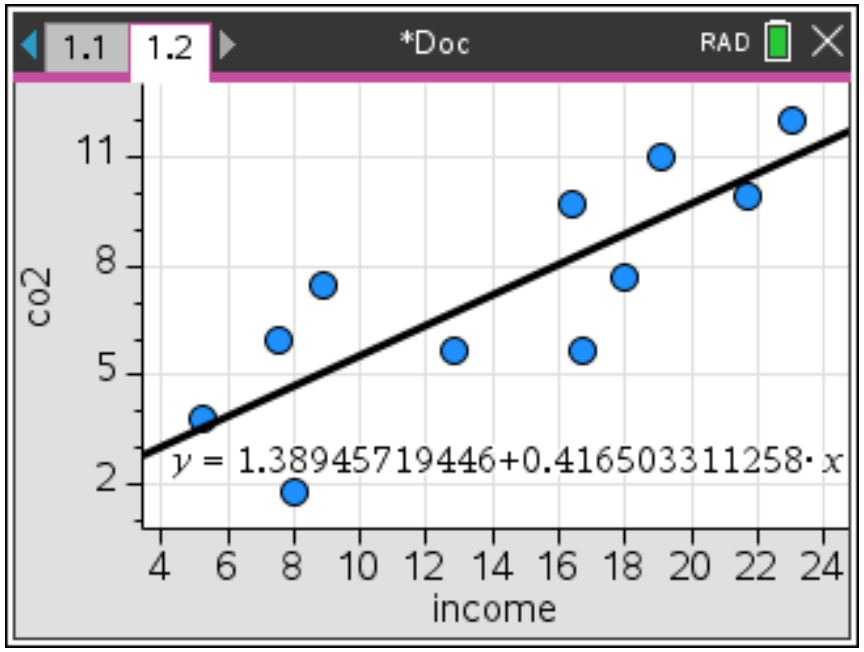


Interpreting the gradient

We can, once again, use a boilerplate sentence to describe the gradient.

$$co2 = 1.39 + 0.417 \times income$$

$y = a + b.x$



The gradient tells us that, **on average**, for every **one-thousand dollars increase in income** there will be a **0.417 tonne increase** in the amount of **C02** pumped into the atmosphere.

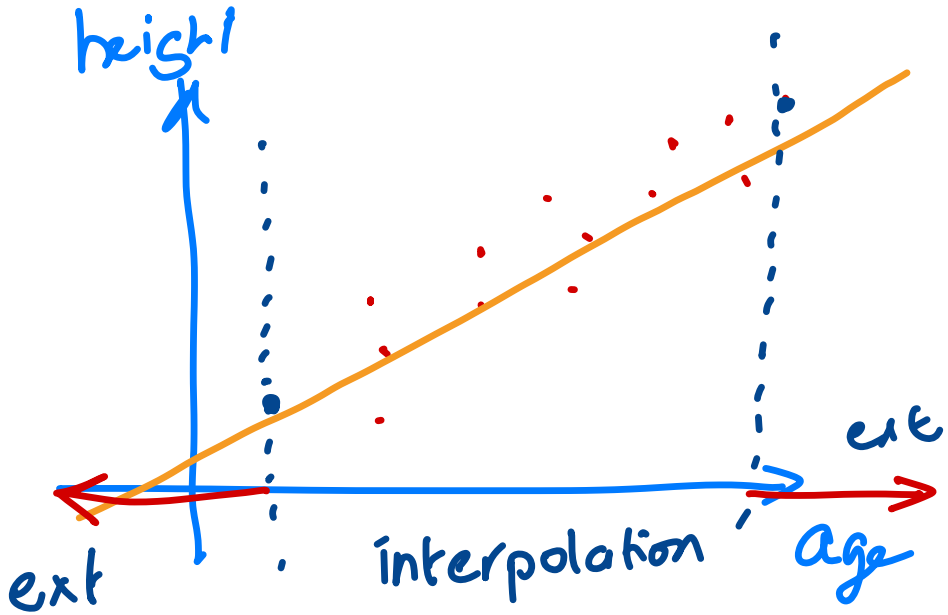


Interpolation or extrapolation

It is important to note that the equation is **only valid** between the points which created it.

If we think about age and height, there will have to be a limit to how tall someone can grow!

So, we need to understand the ideas behind interpolation and extrapolation.

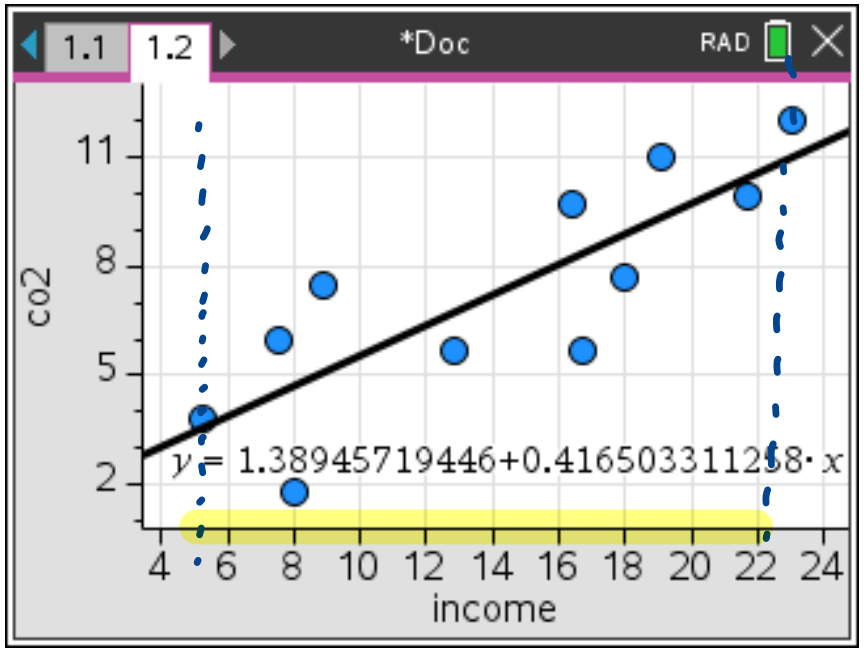




Interpolation

When we are asked to predict an RV for an EV which falls between the highest EV and lowest EV this is called **interpolation**.

We are generally happy that the prediction we will get will be sensible.

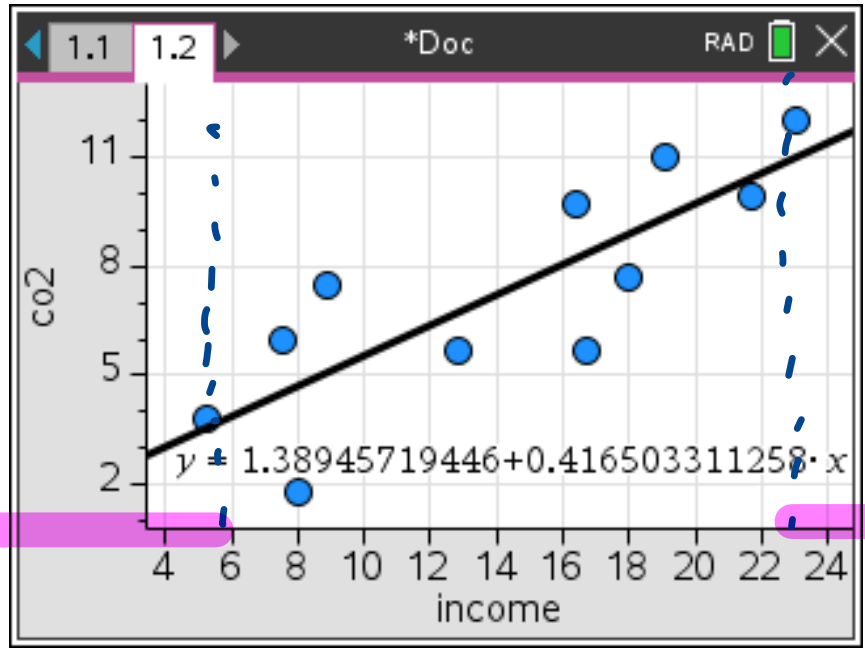




Extrapolation

When we are asked to predict an RV for an EV which falls outside of the highest EV and lowest EV, this is called **extrapolation**.

We do not really like to predict for these values as the model might break down outside of these values.





Example 1

A regression line is used to model the association between the *time*, in hours, a group of students spent studying for an examination and their *mark (%)*. The equation of the regression line is:

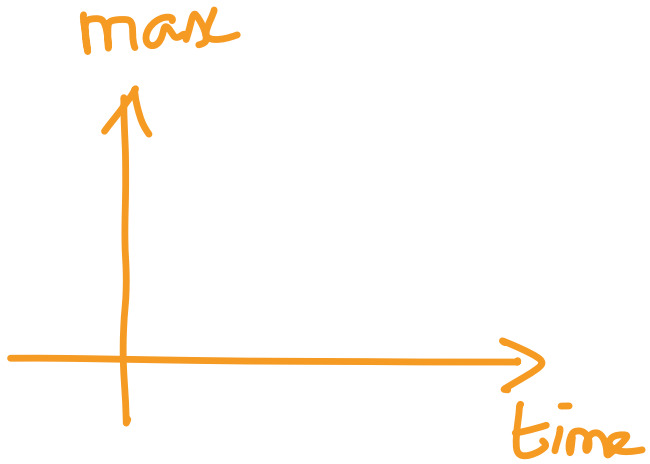
$$mark = 30.8 + 1.62 \times time$$

a **i** Write down the value of the intercept.

ii Interpret the intercept in the context of these variables.

b **i** Write down the value of the slope.

ii Interpret the slope in the context of these variables.



$y = a + b \cdot x$

$y = a + bx$

30.8 ✓

✓

1.62 ✓

the intercept suggests, on average, that for no time in hours spent studying, I could gain an exam score, %, of 30.8.

the slope suggests, on average, for every 1 hour increase in study, my mark, in %, will increase by 1.62



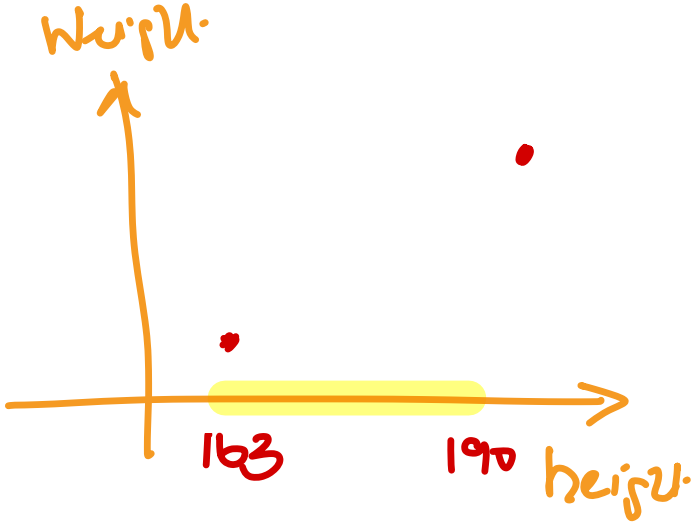
Example 2

The equation relating the weights, in kg, and heights, in cm, of a group of students whose heights ranged from 163 cm to 190 cm, is:

$$weight = -40 + 0.60 \times height$$

Use this equation to predict the weight of students with the following heights.
Are you interpolating or extrapolating?

a 170 cm **b** 65 cm



a. $h = 170$ cm interpolating

$W = 62$ kg

b. $h = 65$ cm extrapolating

$W = -1$ kg



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