The cosine rule

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to use the cosine rule to find the unknown side when given two sides and the angle between them.
- To be able to use the cosine rule to find an angle in a triangle when given the three sides.
- To be able to identify when the sine rule or the cosine rule should be used.

Recap

In the last lesson we looked at the sine rule.

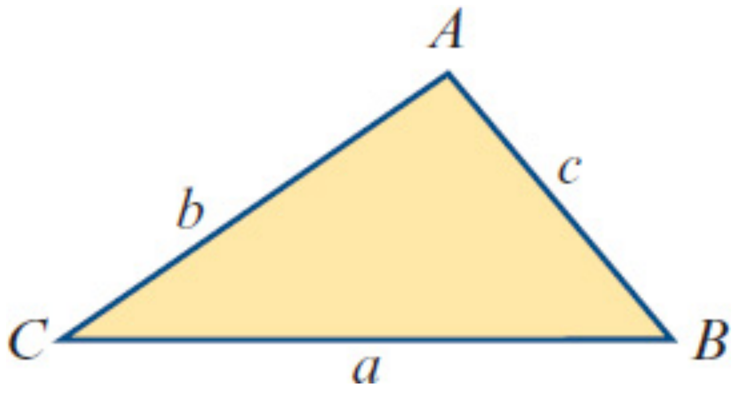
This was used to find the missing sides and/or angles in non-right angled triangles.

We could use the sine rule when we had:

- Two sides and one angle where the angle was opposite one of the sides.
- Two angles and one side where the side was opposite one of the angles.

But what would happen if we didn't have either of the cases above?

Well, there is another rule we can use for just this case!

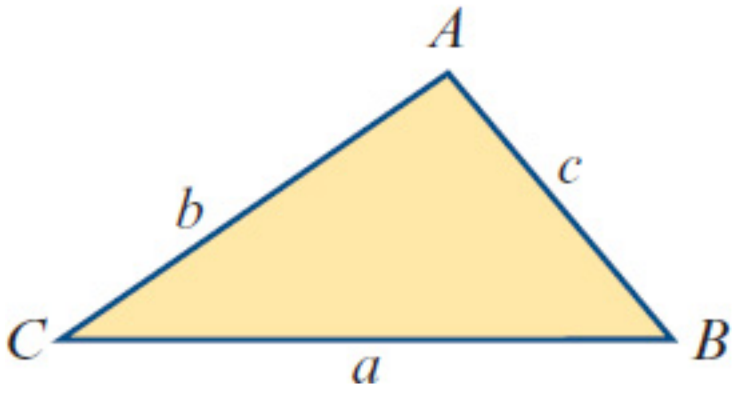


Recap: Naming of sides and angles for non-right angled triangles

Remember:

For right angled triangles, we knew that the sides should have certain letters given to them. This is also true for non-right-angled triangles.

This is also the case for the sides and angles of non-right-angled triangles. We learned about it in the last lesson.



It doesn't really matter which side you call which letter but it is important to make sure the **opposite angle has the same letter but in lower case**.

The cosine rule

The following is the formula we call the **Cosine Rule.**

It is used when we have a triangle and are given:

- All three sides
- Two sides and an included angle

There are two forms of the cosine rule! One helps us find the length of a missing side and the other the size of a missing angle.

$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

I remember that the letter in the front is the same as the angle in the COS.
This is used when we are trying to find a missing side.

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

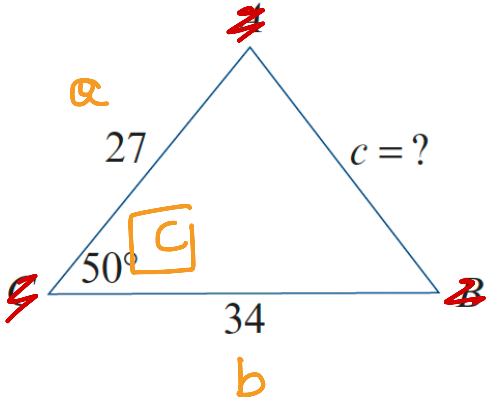
This is used when we are trying to find a missing angle.

Example: The sine rule

$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

Find side c, to two decimal places, in the triangle shown.



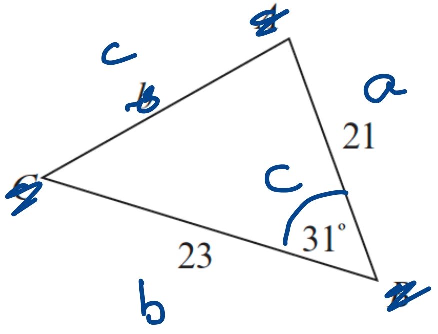
$$c^2 = a^2 + b^2 - 2.a.b.\cos C$$

$$= 27^2 + 34^2 - 2.27.34.\cos(50^\circ)$$

$$c = 26.55$$

Example: The sine rule

Find side b to two decimal places.



$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

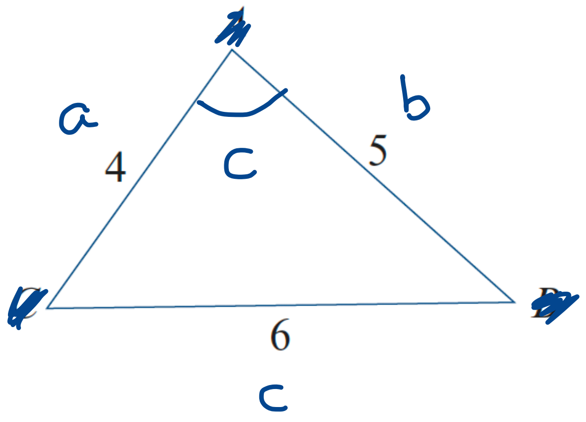
$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

$$c^2 = 21^2 + 23^2 - 2.21.23.\cos(31)$$

$$c = 11.92$$

Example: The sine rule

Find the largest angle, to one decimal place, in the triangle shown.



$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

$$\cos C = \frac{4^2 + 5^2 - 6^2}{2 \cdot 4 \cdot 5}$$

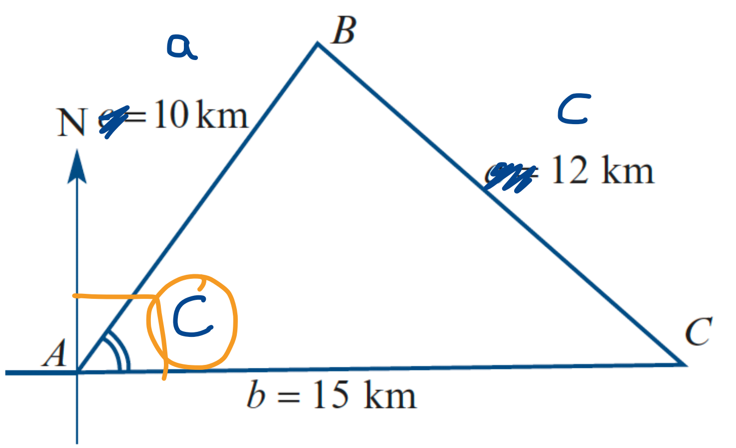
$$C = 82.8^\circ$$

Example: The sine rule

$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

A yacht left point A and sailed 15 km east to point C. Another yacht also started at point A and sailed 10 km to point B, as shown in the diagram. The distance between points B and C is 12 km.



a What was the angle between their directions as they left point A? Give the angle to two decimal places.

b Find the bearing of point B from the starting point, A, to the nearest degree.

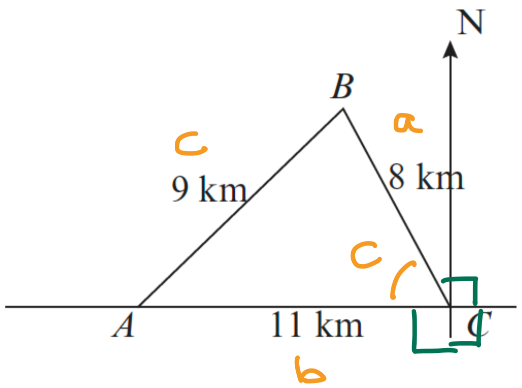
$$\cos C = \frac{10^2 + 15^2 - 12^2}{2 . 10 . 15} \qquad C = \underline{\underline{52.89^\circ}}$$

$$\therefore \text{Bearing} = 90^\circ - 52.89^\circ = 37.11^\circ = 37^\circ$$

$$= \underline{\underline{037^\circ}}$$

Example: The sine rule

A bushwalker left their camp at point C and walked 8 km to point B, as shown in the diagram. A friend walked 11 km to point A, a distance of 9 km from B.



$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

a What was the angle between their directions as they left C? Answer to one decimal place.

b What was the bearing of point B from their starting point, C, to the nearest degree?

$$\cos(C) = \frac{8^2 + 11^2 - 9^2}{2 . 8 . 11}$$

$$C = 53.8^\circ$$

Bearing $= 90 + 90 + 90$

$+ 54^\circ$

$= 324^\circ$

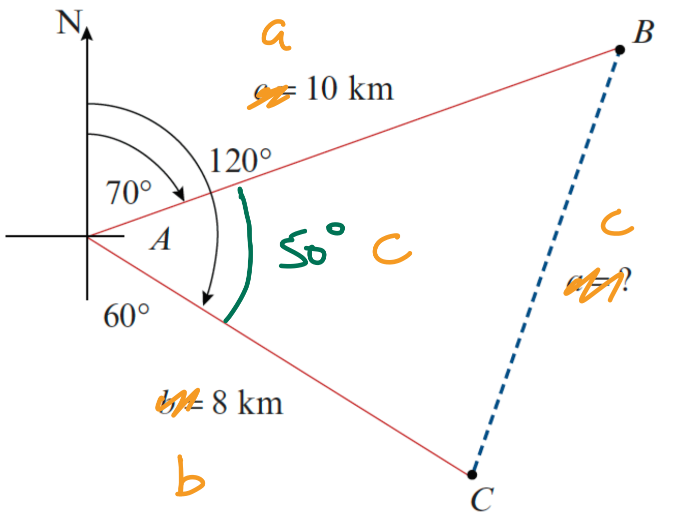
Example: The sine rule

A bushwalker left his base camp and walked 10 km in the direction 070°.

His friend also left the base camp but walked 8 km in the direction 120°.

a Find the angle between their paths.

b How far apart were they when they stopped walking? Give your answer to two decimal places.



$$c^2 = a^2 + b^2 - 2 \times a \times b \times \cos(C)$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2 \times a \times b}$$

$120^\circ - 70^\circ = 50^\circ$

$c^2 = 10^2 + 8^2 - 2.10.8.\cos(50)$

$c = 7.82$ km

Questions to complete

The following questions represent the minimum I am asking you to complete. The more questions you do, the better you will be at Mathematics.

Exercise: 11H The Cosine Rule

Questions:
4abc, 5, 8abc, 10, 13, 14

Extension Questions:
15, 17

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