The sine rule

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to use the sine rule to find an unknown angle, given two sides and an opposite angle.
- To be able to use the sine rule to find an unknown side, given two angles and a side.
- To be able to find the required angles and sides when the given information fits two possible triangles.

Recap

This chapter has been about everything we can possibly know about triangles.

We have looked at how we can use Trigonometry to find the size of missing sides and angles (when we are given the right information). We have looked at how we can apply this knowledge to real world problems.

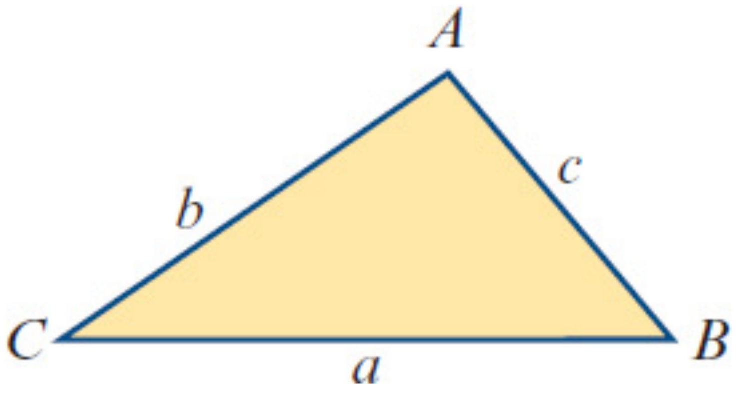
We have looked at how it applies to angles of elevation and depression and, in the last lesson, how we can apply it to bearings and navigation.

All this work has looked at one type of triangle; the right angled triangle.

But, we know that not all triangles are right angled. So, how would we find the size of missing angles and sides for these types of triangles?

Naming of sides and angles for non-right angled triangles

For right angled triangles, we knew that the sides should have certain letters given to them. This is also true for non-right angled triangles.



It doesn't really matter which side you call which letter but it is important to make sure the **opposite angle has the same letter but in lower case**.

In the UK we tended to have it drawn like this …

The sine rule

The following is the formula we call the **Sine Rule.**

It is used when we have a triangle and are given:

- Two sides and one angle (where one side is opposite the given angle)
- Two angles and one side (where the side is opposite one of the given angles)

The side is always divided by the sine of it's angle.

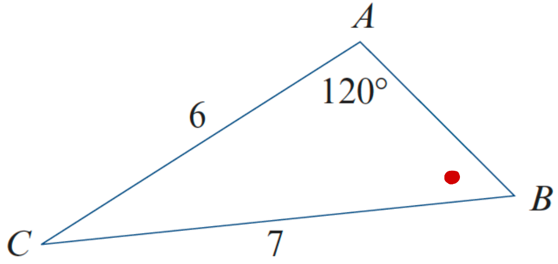
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Note: We only every use two parts of the equation.

Example: The sine rule

It's probably easier to show some examples of how to use it.

Find angle B in the triangle shown to one decimal place.



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$\text{nSolve}\left(\frac{7}{\sin 120} = \frac{6}{\sin B}, b\right)$$

F: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

S: $$\frac{7}{\sin 120^\circ} = \frac{6}{\sin B}$$

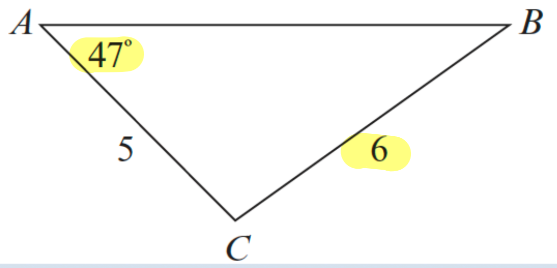
$$B = 47.9^\circ$$

Example: The sine rule

It’s probably easier to show some examples of how to use it.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Find angle B to one decimal place.



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{nSolve}\left(\frac{6}{\sin 47} = \frac{5}{\sin B}, b\right)$$

$$\frac{6}{\sin 47} = \frac{5}{\sin B}$$

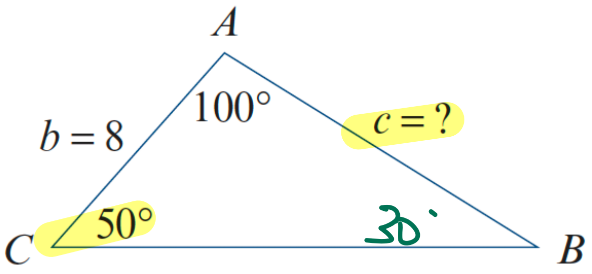
$$\therefore B = 37.6^\circ$$

Example: The sine rule

It's probably easier to show some examples of how to use it.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Find side c in the triangle shown to one decimal place.



$$\frac{c}{\sin 50} = \frac{8}{\sin 30^\circ}$$

$$\text{nSolve}\left(\frac{c}{\sin 50} = \frac{8}{\sin 30}, c\right)$$

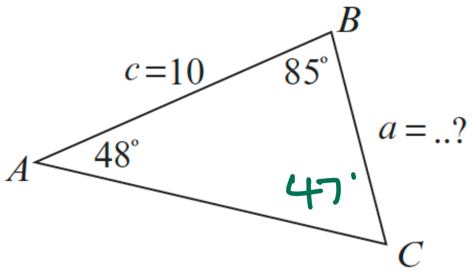
$$c = 12.3$$

Example: The sine rule

It's probably easier to show some examples of how to use it.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Find side a to one decimal place.



$$\frac{10}{\sin 47^\circ} = \frac{a}{\sin 48^\circ}$$

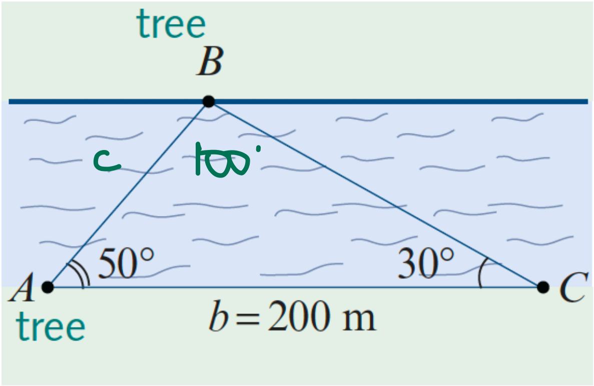
nSolve(.... , a)

$a = 10.2$

Application: The sine rule

Leo wants to tie a rope from a tree at point A to a tree at point B on the other side of the river. He needs to know the length of rope required.

When he stood at A, he saw the tree at B at an angle of 50° with the riverbank. After walking 200 metres east to C, the tree was seen at an angle of 30° with the riverbank.



$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Find the length of rope required to reach from A to B to two decimal places.

$$\frac{c}{\sin 30^\circ} = \frac{200}{\sin 100^\circ}$$

$$\therefore c = \underline{\underline{101.54}}\text{ m}$$

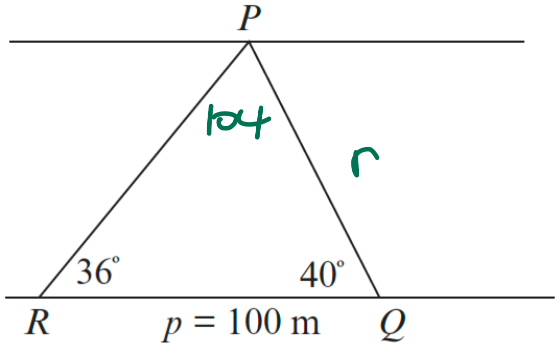
Application: The sine rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Engineers needed to construct a bridge across a canyon from P on the edge of one side to Q on the edge of the other side. The diagram is the view looking down on the parallel sides of the canyon and the proposed position of the bridge, PQ.

From point R on one side of the canyon, a surveyor sighted post P on the other side at an angle of 36° to the edge of the canyon. Moving 100 m to Q, she sighted point P at an angle of 40° to the edge.

Find the required length of the bridge to two decimal places.



$r = 60.58\text{m}$

$$\frac{r}{\sin 36} = \frac{100}{\sin 104}$$

The sine rule: Ambiguous Case

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

In triangle ABC, A=40°, c=9 cm and a=6 cm. Find angle A.

Note: There are actually two triangles which can be drawn.

There is an easier way to do these questions!

When they ask you to find an angle, we do a check to see if more than one answer exists.

Questions to complete

The following questions represent the minimum I am asking you to complete. The more questions you do, the better you will be at Mathematics.

Exercise: 11G The Sine Rule

Questions:
4, 5, 6ade, 7, 8, 9a, 10, 11, 12, 18, 19

Extension Questions:
16

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