Similar solids

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to determine when two solids are similar, and to calculate their scale factor.

Recap

The final part of this chapter looks at similarity of 3D shapes. Previously we have looked at the similarity of 2D shapes. Firstly, similar figures and then similar triangles.

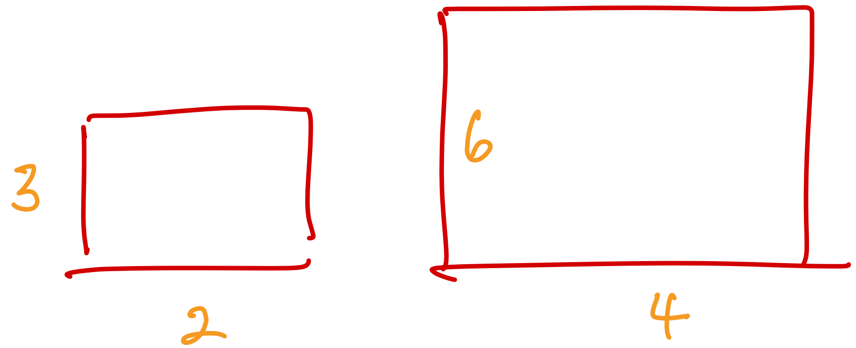
With similar figures we found that we could find a ratio of side lengths and, once we know this, we can find the ratio of areas; as:

$$(ratio\ of\ side\ lengths)^2 = ratio\ of\ areas.$$

With similar triangles, we had three rules we needed to understand and apply; AA, SAS and SSS.

×2 ×2

×4



Similar solids

A solid is a 3D shape.

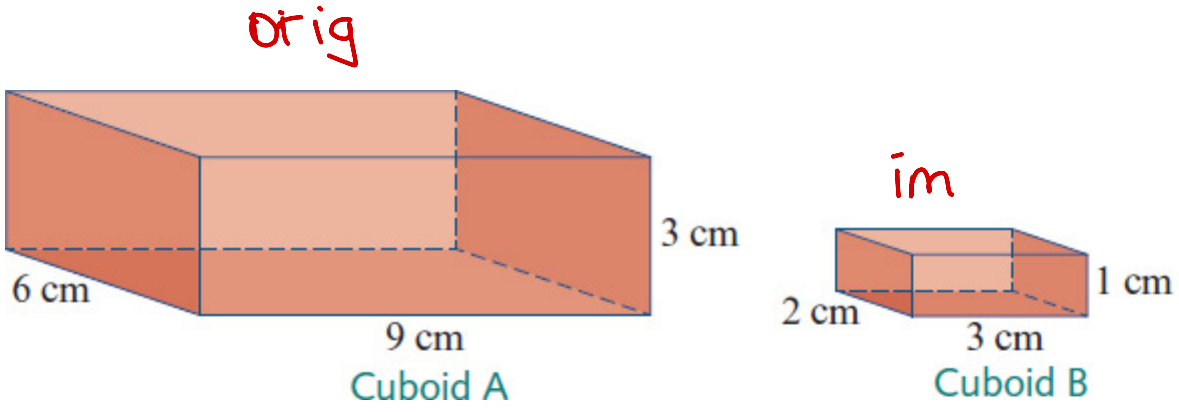
In the same way as we did before, we can find the ratio of the side lengths of 3D objects.

We need to choose similar edges and do the image divided by the original.

We need to check that the ratios are the same for all side lengths.

$$Ratio\ of\ side\ lengths = \frac{side\ length\ of\ image}{side\ length\ of\ original}$$

$$= \frac{3}{9} = \frac{1}{3}$$



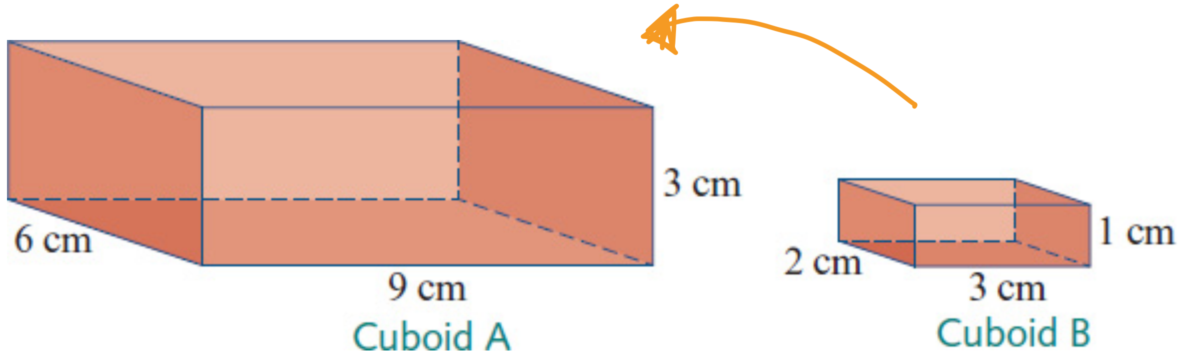
Which is the image and this is the original?
Choose similar side lengths.
Divide them.
Check all ratios are the same.

Relating the volumes of similar shapes

In the previous lesson we found that there was a relation between the ratio of side lengths and the ratio of the areas.

I wonder if there is a similar relationship between the ratio of side lengths and the volumes of solids?

For the following shapes we found that the ratio of side lengths was $\frac{1}{3}$ … lets find the volumes of each shape and then the ratio.



$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

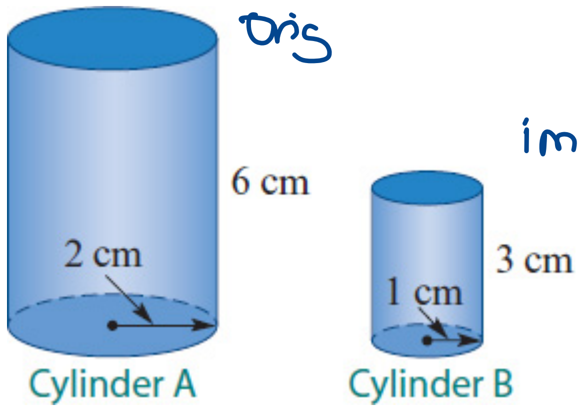
×3
×3 = 27
×3

Does this work with all shapes?

Let's have a look at cylinders.

Let's find the ratio of side lengths.

Then find the volumes and see if we can find the ratio of the volumes and see what happens.



orig

im

$V = \pi r^2 h$

ratio side lengths $= \frac{3}{6} = \frac{1}{2}$

ratio of volumes $= \left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

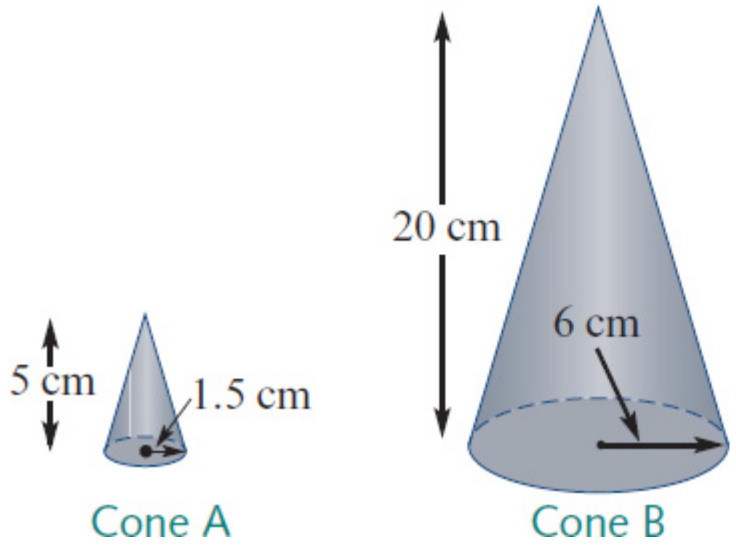
What about cones?

What about cones?

Let' find the ratio of side lengths and then the ratio of the volumes.

Remember the volume of a cone is:

$$Volume = \frac{1}{3} \times \pi \times r^2 \times h$$



$V_A = \frac{1}{3} \cdot \pi \cdot 1.5^2 \cdot 5$

$= 11.78\ cm^3$

$V_B = \frac{1}{3} \cdot \pi \cdot 6^2 \cdot 20$

$= 753.98\ cm^3$

$\therefore$ sfe v = 64

rsl $= \frac{20}{5} = 4$

rv $= (4)^3 = 4 \times 4 \times 4$

$= 64$

Bringing it all together

So, it seems to be that if we know the ratio of side lengths then …

For 2D shapes:

$$Ratio\ of\ areas\ =\ (ratio\ of\ side\ lengths)^2$$

For 3D shapes:

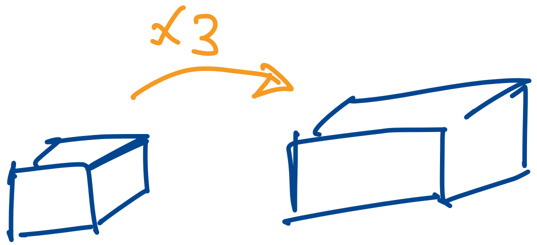
$$Ratio\ of\ areas\ =\ (ratio\ of\ side\ lengths)^3$$

Why do we need to know this?

Well, if we know the ratio of sides and the area of volume of one shape, we can find the area of volume of an image.

Example

Two solids are similar such that the larger one has all of its dimensions three times that of the smaller solid. How many times larger is the larger solid's volume?



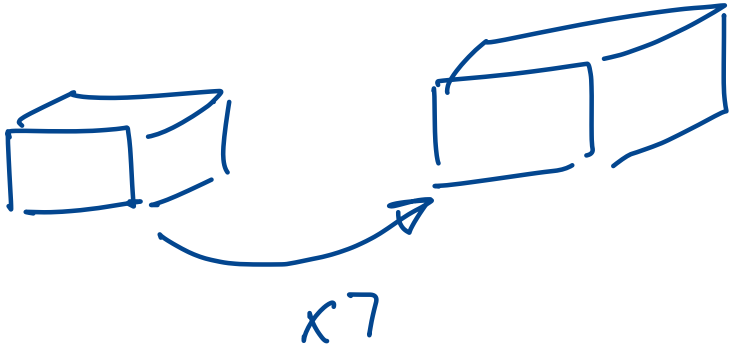
ratio of side lengths = 3

ratio of volumes = $(rsl)^3 = 3^3 = 3 \times 3 \times 3$

$= 27$

Example

A solid has dimensions seven times those of a smaller similar solid. How many times is the volume of the larger solid greater than the volume of the smaller solid?



$$rv = (rsl)^3$$

$$rsl = \sqrt[3]{rv}$$

$$rsl = 7$$

$$\therefore rv = (7)^3 = 7 \times 7 \times 7 = \underline{\underline{343}}$$

Work to complete

The work I am asking to be completed for this topic is shown below.

This is the minimum work which should be completed. The more questions which are answered the better your chance of success in exams. Questions towards the end of the exercises and in the Chapter Review are the best practice you can do.

Questions to complete:
Exercise 10J: All questions