Volume

Year 11 General Maths
Units 1 and 2

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- To be able to determine the volumes of rectangular prisms, triangular prisms, square prisms, cylinders and cones.
- To be able to find the capacity of three-dimensional containers

Recap

As we continue through the section relating to measurement, scale and similarity for the Year 11 General Maths course we have spent a lot of time looking at areas and perimeters of shapes. The last lesson looked at how we can find the areas of sectors and the lengths of arcs.

All the work we have been doing so far relates to 2D shapes.

But what is really 2D in life?

Let’s ramp this up and look at some shapes which are 3D.

Question: How many chocolates can fit inside a standard box of celebrations?

Finding the space inside a shape

When we are asked to find how much space or air or liquid which can fit into a 3D object, we are really being asked to find the volume of the shape.

There are LOTS of formulae which we can write in our summary books, and the Cambridge textbook has many of them already for us.

Once we know which shape we have, we match the formula and then put in the values we have been given.

Common shapes we will need to find the volumes for are shown on the right.

Common shapes:

Rectangular prisms
Square prisms
Triangular prisms
Cylinders
Spheres

Volume and capacity

Lots of students get confused when being asked to find the capacity of something.

To be honest, it's Barry being a pain again.

When we are asked to find the capacity of something, we are being asked to find the how much liquid can be fit into something. So … volume by another name!

There are important units we need to use and there are some conversions we need to write into our summary books.

Volume to capacity conversion

The following conversions are useful to remember.

$$1 \text{ m}^3 = 1000 \text{ litres (L)}$$
$$1 \text{ cm}^3 = 1 \text{ millilitre (mL)}$$
$$1000 \text{ cm}^3 = 1 \text{ litre(L)}$$

What is a prism?

When I worked at Tesco's in the UK, they had the most amazing machine. I wanted to spend all day working with it … I was fascinated.

Or easily pleased … but that's another story!

A bread cutting machine!!

We put the bread into the machine and out comes the bread cut into perfectly matching pieces. The same shape and size.

Magic!

This is pretty much the definition of a prism.
If you can put it into a bread slicing machine in a certain way, and all the pieces come out looking the same then the shape is a prism.

Examples of prisms

It always comes down to food!

The Toblerone is a triangular prism.
The Jelly Tots package is a cylinder.

The best chocolate on the planet

The quickest sugar high you will ever have!

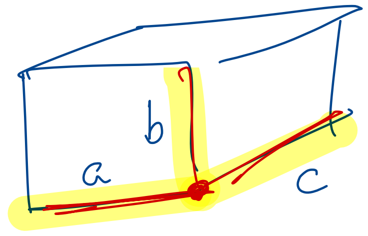
Finding the volume

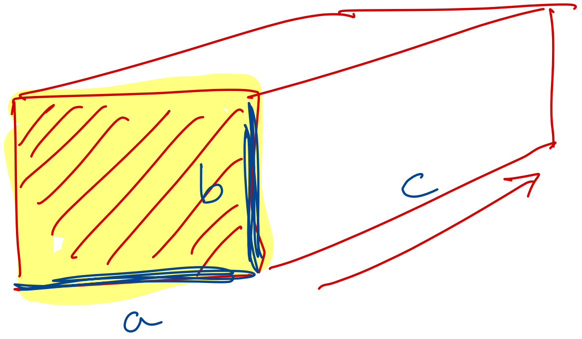
$V = abc$

There are lots of ways to find the volumes but one of the best ways it to know about something called the "**cross sectional area**"

When we cut through something (or look at the front of the shape) we can see that we have a cross section. We can find the area of this and then multiply it by how far back to goes to find the volume.

The best way to see this is using a cube.





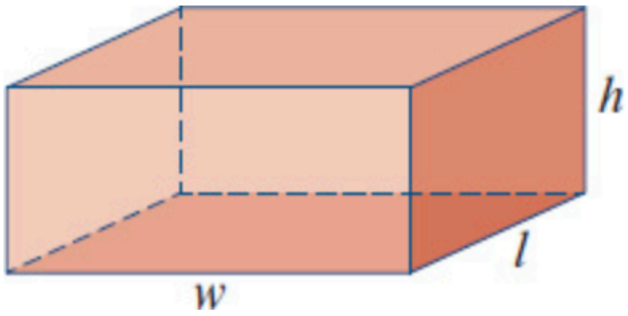
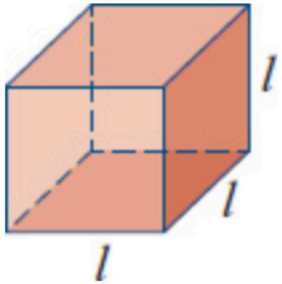
Cross Sec Area $= ab$

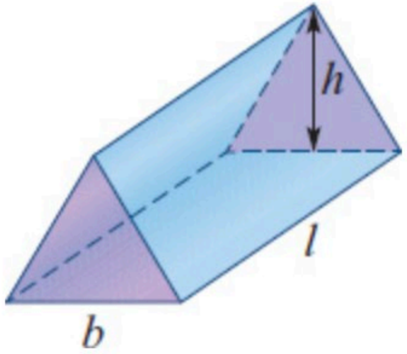
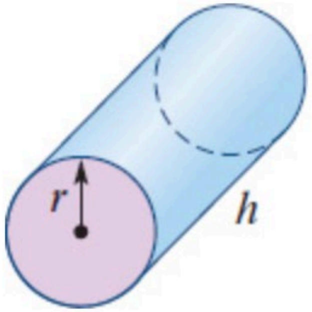
$V = X_s \times c$

$= ab \times c = abc$

Common shapes and their volumes

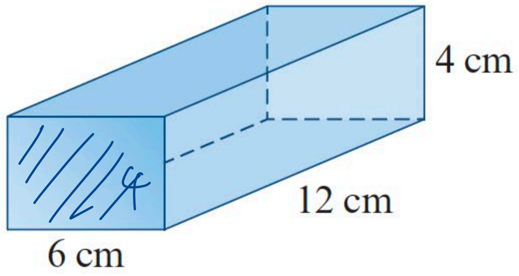
Here are some of the more common formulae which you will be expected to use:

Shape	Volume
Rectangular prism (cuboid) 	$V = lwh$
Square prism (cube) 	$V = l^3$

Shape	Volume
Triangular prism 	$V = \frac{1}{2}bhl$
Cylinder 	$V = \pi r^2 h$

Examples

Find the volume of the cuboid shown.

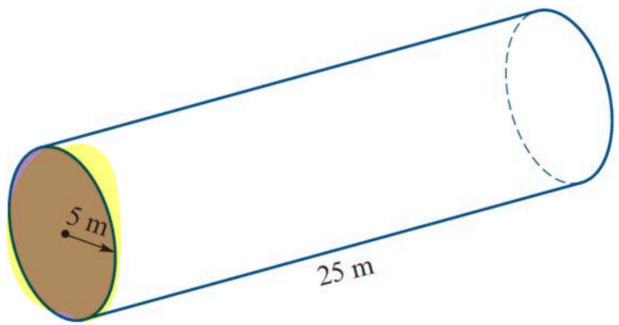


$$\text{Volume} = 6 \times 4 \times 12$$

$$= 288 \text{ cm}^3$$

Examples

Find the volume of this cylinder in cubic metres.
Give your answer to two decimal places.

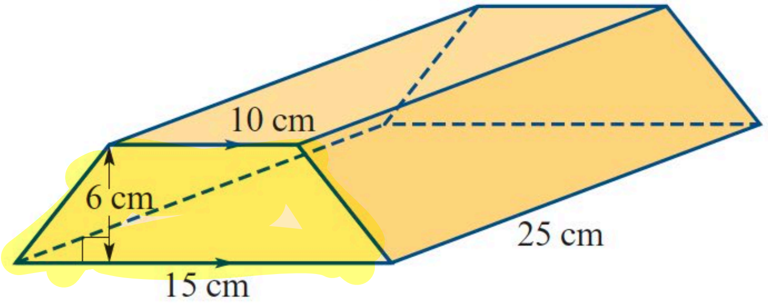


$V = \pi \times 5^2 \times 25$

$= 1963.5 \, m^3$

Examples

Find the volume of the three-dimensional object shown.



Hint:
With this example it might be easier to find the area of cross section (the front) and then multiply it by how far back it goes.

The front is a trapezium!

$$V = \frac{(10+15)\times 6}{2} \times 25$$

$$= 1875\ cm^3$$

Example: Find the capacity

A drink container is in the shape of a cylinder. How many litres of water can it hold if the height of the cylinder is 20 cm and the diameter is 7 cm? Give your answer to two decimal places.

Remember: Capacity is just the volume of something but measured in litres

Volume to capacity conversion

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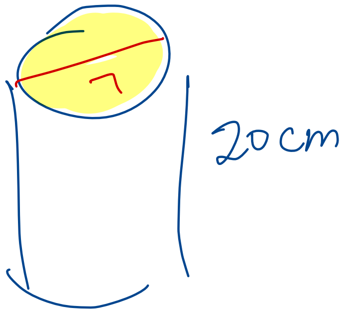


$$V = \pi \times r^2 \times 20$$
$$= \pi \times (3.5)^2 \times 20$$
$$= 769.69 \text{ cm}^3$$

$$\therefore \text{cap} = 769.69 \div 1000$$
$$= 0.77 \text{ L}$$

Ice cream should come in tubs not cones

Nobody should be allowed to sell these things. They are gross!! WHY??

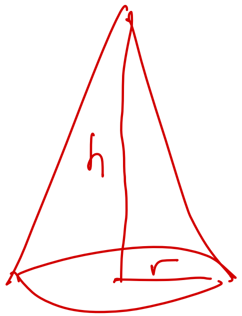
Ice cream should come in old fashioned cones.

The humble cone is another shape which we are going to need to find the volume of.

It has a great little formula:

$$Volume\ of\ a\ cone = \frac{1}{3}\pi r^2 h$$

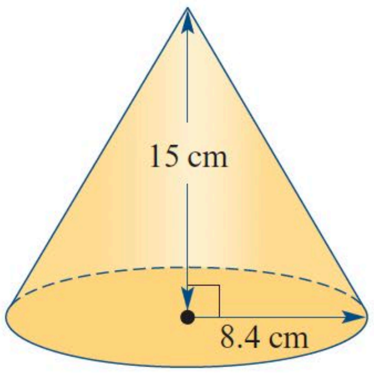
Where ‘r’ is the radius of the circular bit and ‘h’ is the height of the cone



Example

Find the volume of this right circular cone.
Give your answer to two decimal places.

Note: A right circular cone is one where we can draw a perpendicular line from the centre of the circle to the very top.



$$V = \frac{1}{3} \times \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \pi \times 8.4^2 \times 15$$

$$= 1108.35 \text{ cm}^3$$

The correct shape balls

A real football is **spherical** in shape.

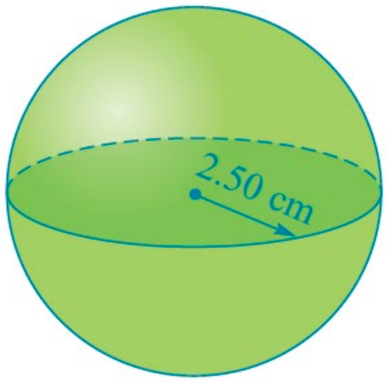
We can find the volume of a sphere using another formula (which must be written in your summary book):

$$Volume\ of\ a\ sphere = \frac{4}{3}\pi r^3$$

NOT …

Example: Finding the volume of a sphere

Find the volume of this sphere, giving your answer to two decimal places.



$$V = \frac{4}{3} \times \pi \times r^3$$

$$= \frac{4}{3} \times \pi \times 2.5^3$$

$$= \underline{65.45} \text{ cm}^3$$

Work to complete

The work I am asking to be completed for this topic is shown below.

This is the minimum work which should be completed. The more questions which are answered the better your chance of success in exams. Questions towards the end of the exercises and in the Chapter Review are the best practice you can do.

Questions to complete:
Exercise 10E: 1acdegjk, 2, 4, 6, 7, 8, 9, 13, 14, 17