Further symmetry properties and the Pythagorean Identity

Sunday, 25 March 2018 9:06 pm

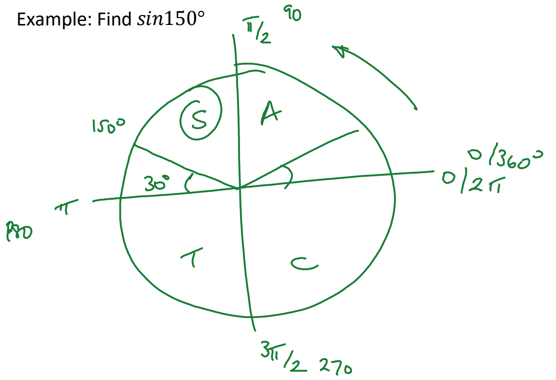
Work to be completed at the end of teaching:

Pythagorean Identity	6C	2	

RECAP:

Previously we looked at the fact that we can use the Unit Circle to help us find values of $sin\theta, cos\theta$ *and* $tan\theta$ for angles between 0° and 360°.
We used ASTC to help us realise that, given a reference angle, we can use this to find angles in other quadrants:

Example: Find $sin150°$



$\sin 150° = +\sin 30°$

$= \frac{1}{2}$

Other ways to find angles

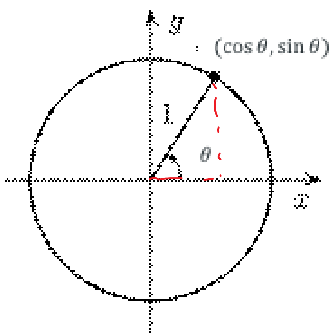
Remember, Pythagoras ...

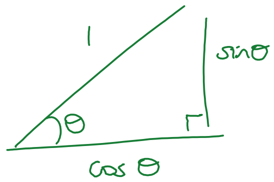
People think he came up with the famous $a^2 = b^2 + c^2$

$c^2 = a^2 + b^2$

It's possible he didn't.
He created his own religion.
He hated people who questioned him.
He (allegedly) murdered someone.

The theory he "badged" we use today and can be applied to the Unit Circle





$c^2 = a^2 + b^2$

$1^2 = (\sin\theta)^2 + (\cos\theta)^2$

$\sin^2\theta + \cos^2\theta = 1$

Using the unit circle and Pythagoras' Theorem we can see that:

What does this have to do with the price of fish?

Nothing ... but apparently there are things which have everything to do with the price of fish.

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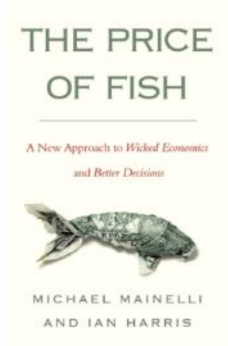
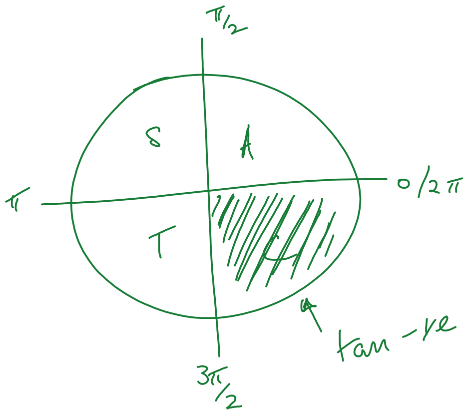
We can use his theorem and the knowledge we have to answer some pretty excellent questions:



Given that $\cos x = \frac{3}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, find $\sin x$ and $\tan x$.



$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{3}{5}\right)^2 = 1$

$\sin^2\theta + \frac{9}{25} = 1$

$\sin^2\theta = 1 - \frac{9}{25}$

$\sin^2\theta = \frac{16}{25}$

$\sin\theta = \pm\sqrt{\frac{16}{25}}$!

$\sin\theta = \pm \frac{4}{5}$

$\therefore \sin\theta = -\frac{4}{5}$

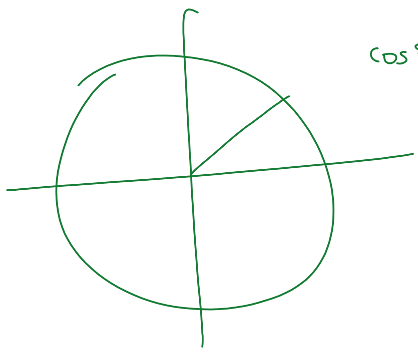
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= -\frac{4}{5} \div \frac{3}{5}$

$= -\frac{4}{5} \times \frac{5}{3}$

$= -\frac{4}{3}$

There is, of course an easier way!



$\cos\theta = \frac{3}{5}$

$\theta = \cos^{-1}(3/5)$