# Families of quadratic polynomial functions

Thursday, 8 March 2018 6:02 pm

Work to be completed at the end of teaching:

| Families of quadratic functions | MM Ex. 3K | 1 3 5 6 8 9 10 11 13 14 16 17 19 20 21 |
|---|---|---|

**RECAP:**

We have almost reached the end of the work on Quadratics.
So far we have learnt how to:

- Expand and collect like terms
- Factorise quadratics using a number of methods:
  - T Method
  - Cross Method
  - Quadratic Formula
  - Completing the square
- Graph Quadratics
- Solve quadratic inequalities
- Use the discriminant to find how many solutions there are to a problem
- Solve simultaneous equations involving quadratics and straight lines.

Importantly, everything you have learnt here can be applied to all areas of Methods 1 and 2. When presented with a question you need to identify what it is asking you to achieve, then choose the right set of tools to reach your answer.

**Doing things backwards**

Most of the questions we have been doing so far relate to solving quadratics or finding points of intersections.

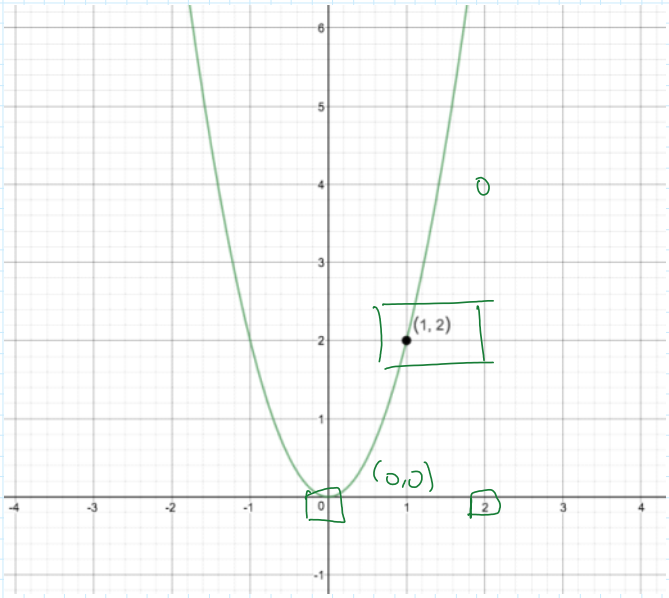
Now we are going to be looking at how to find the equation of a quadratic when given certain information:

**Remember: All quadratics come from $y = x^2$**

$y = ax^2 + bx + c$

$y = 3x^2 - 4x + 7$

**Case 1: Standard Form**




Dilation

$y = ax^2$

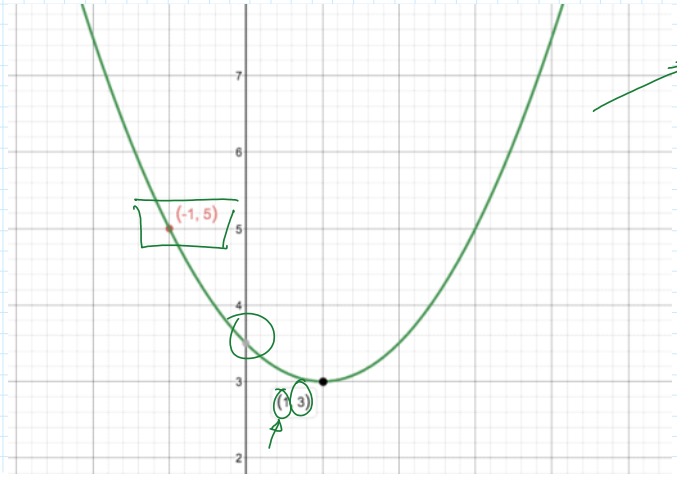
$2 = a(1)^2$

$2 = a$

$y = 2x^2$

This graph is the **standard quadratic** which has been dilated.
We need to be given at least one point to help us find the value of $a$.

**Case 2: Turning point form**




$y = a(x - b)^2 + c$

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$y = a(x - 1)^2 + 3$

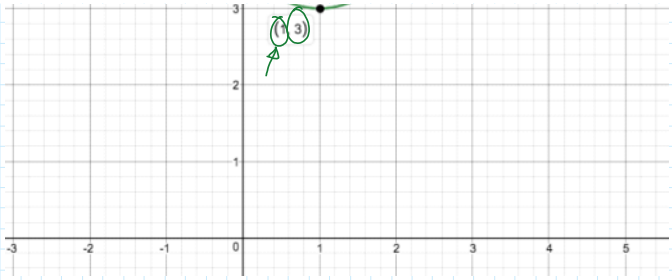
$5 = a(-1 - 1)^2 + 3$

$5 = a(-2)^2 + 3$

$2 = a4$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x - 1)^2 + 3$



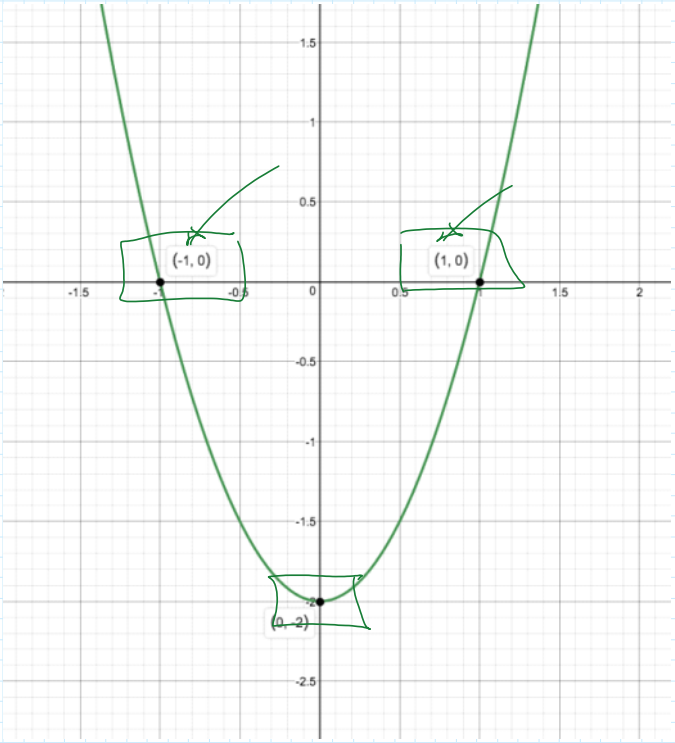

$2 = a4$

$a = \frac{1}{2}$

$y = \frac{1}{2}(x-1)^2 + 3$

As this graph has been given in **turning point form**, we can use the above equation to help us find the value of $a, b$ *and* $c$.
With the graph shown, we would only really need one co-ordinate (outside of the turning point) to find the value of $a$.

### Case 3: Intercept form




$y = a(x+b)(x+c)$ Linear Factors

$x = -1$
$x = 1$
$x+1 = 0$
$x-1 = 0$

$y = a(x+1)(x-1)$

$-2 = a(1)(-1)$

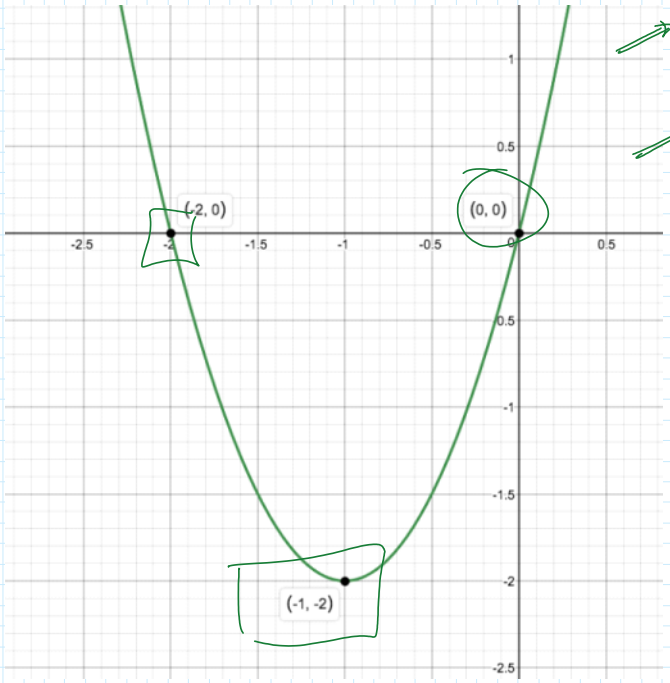
$-2 = -a$

$a = 2$

$y = 2(x+1)(x-1)$

When given two intercepts, we can use the above form to help us find the quadratic equation. With this form, you need to be given an additional co-ordinate to help you find the value of $a$.

### Case 4: One intercept at the origin




$y = ax^2 + bx$

$-2 = a(-1)^2 + b(-1)$

$-2 = a - b$

This can be re-written as:

$y = x(ax+b)$

$y = (x+0)(ax+b)$

$(x+0)(ax+b) = 0$

$x+0 = 0$ or $ax+b = 0$

$ax + b = 0$

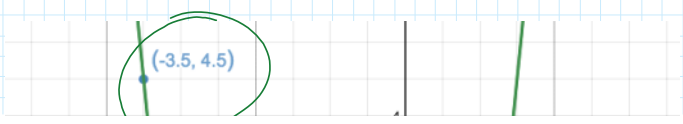
$a(-2) + b = 0$

$-2a + b = 0$

$a - b = -2$

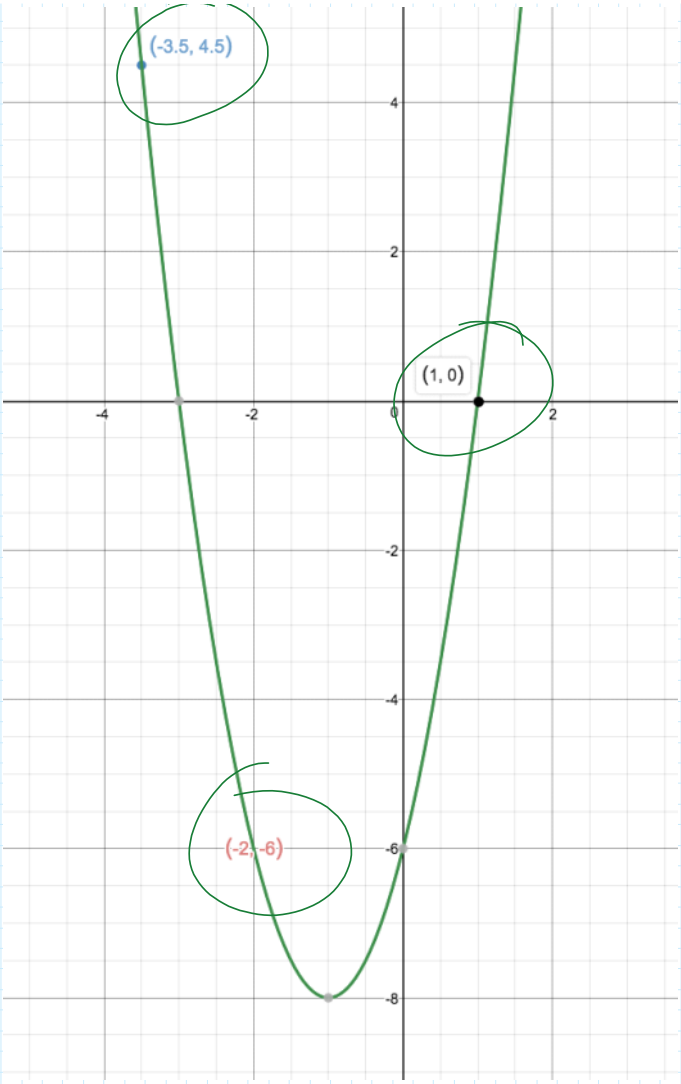
Here we can see that one intercept is given.
We need an additional point to help us find the missing pronumerals.

### Case 5: Three points given




$y = ax^2 + bx + c$

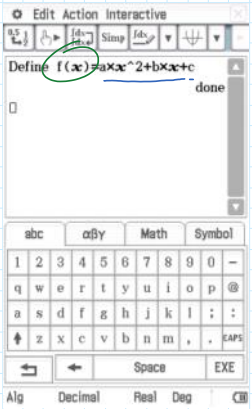
In this case we would use our CAS ...




$y = ax^2 + bx + c$

In this case we would use our CAS …

1. DEFINE the standard quadratic function
   a. remember that $y =$ can be replaced with $f(x) =$




$y = ax^2 + bx + c$

$f(x) = ax^2 + bx + c$

$(0,1)$

$(3,6)$

$(9,12)$

$f(0) = 1$

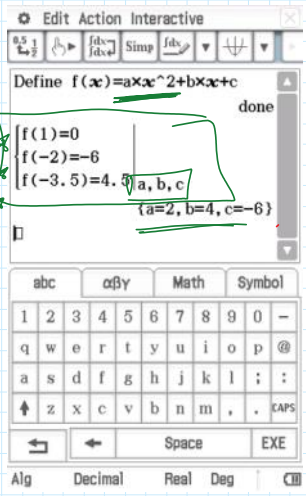
$f(3) = 6$

$f(9) = 12$

2. Remember that we can write co-ordinates in a different form the calculator can now understand:

$(1, 0)$ => $f(1) = 0$
$(-2, -6)$ => $f(-2) = -6$
etc

3. Get your calculator to solve a system of equations:



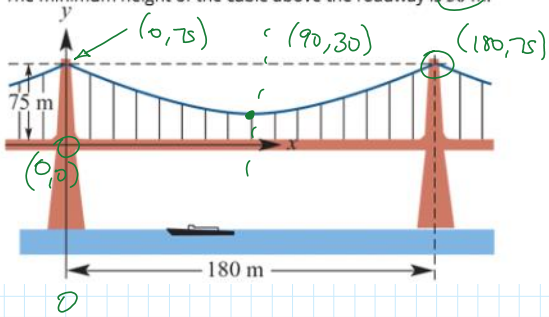

(1,0)
(-2,-6)
(-3.5, 4.5)

x2

$y = 2x^2 + 4x - 6$

✱ **A taste of Methods 3 and 4 …**

**11** Assuming that the suspension cable shown in the diagram forms a parabola, find the rule which describes its shape.
The minimum height of the cable above the roadway is 30 m.




$y = ax^2 + bx + c$

(0,75)
(90,30)
(180,75)

$y = ax^2$

$y = a(x-b)^2 + c$

$y = a(x+b)(x+c)$