

Weighted graphs and networks

**Year 12
Further Mathematics**

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"This is fantastic! Especially in these current circumstances. You just saved me from making this exact video for my students."

Youtube comment
(from a current Teacher)

"Your british accent is so intriguing"

Youtube comment (Other YouTuber)

"Very helpful. I'm having trouble with mathematics during quarantine, I've found this channel off a tiktok one of your students has made and it's much easier now!"

Youtube comment
(from current Year 12 student)

"Mate. You're bloody awesome. To be a maths teacher and to make this stuff.

You know... brilliant."

Youtube comment
(from current Year 12 student)

"Thank you so much for your videos! Especially now that we have had to move to remote learning they have been a life saver!!"

Youtube comment
(from current Year 12 student)

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Further Mathematics course.

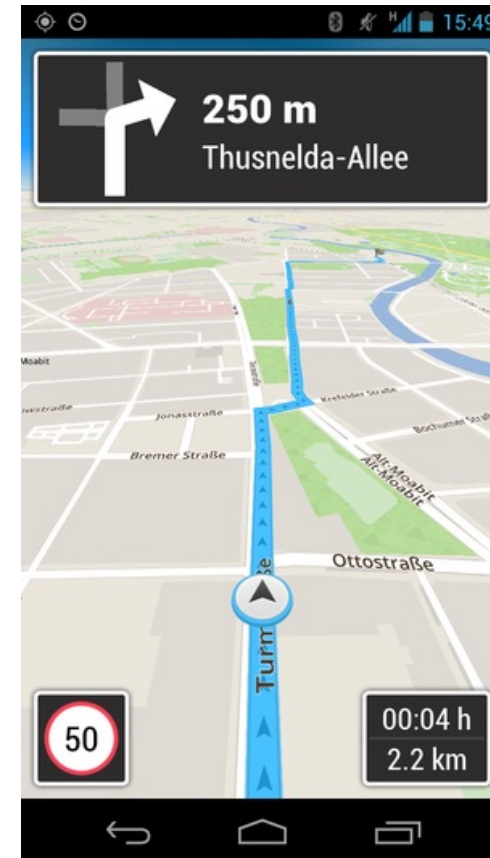
- Understand what it means for a graph to be weighted
- Understand what it means by "shortest path problems"
- Know how to apply Dijkstra's algorithm

Recap of past learning

In previous lessons we have looked at drawing graphs and how they are effectively used to show connections between vertices. In all the examples we have been looking at the edges didn't have any numerical data associated with them. This isn't really modelling a real world situation.

Towns are connected by roads which will have a distance associated with them. Computers connected on the internet have a time taken for transmission of information.

This lesson is going to look at how we can use graphs with numbers to help us solve some real world problems.



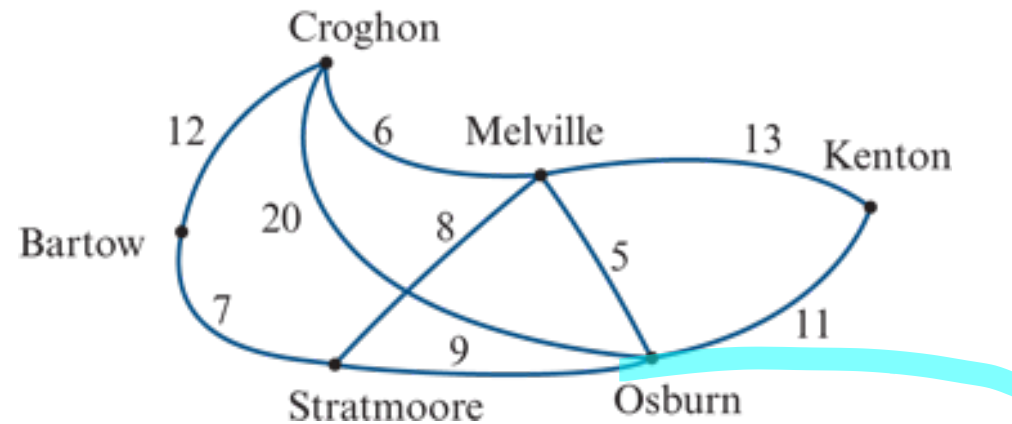
An example of a weighted graph

Here is a graph with some numbers! This is an example of a **weighted graph**.

It would make sense to assume that the numbers represent the distances between each of the towns represented in the graph.

What if we wanted to find the shortest path between two different towns? It always amazes me that the GPS does this so quickly! There must be some sort of algorithm to allow it to do that!

We are going to look at how to do this using the methods available to us in Further Maths,.



Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

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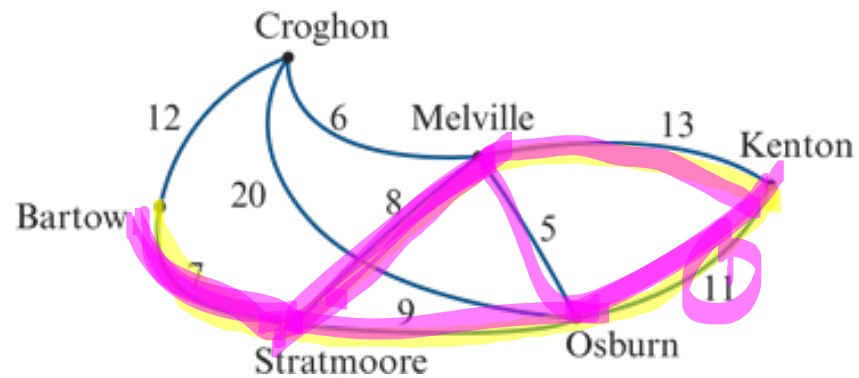
Shortest path

In networks that have only a few vertices, it is often easy to find the shortest path between two vertices by inspection.

Example: Find the shortest path from Bartow to Kenton in the network shown on the right.

Steps to solve this:

- List all the routes
- Add all the distances
- Find the shortest one!



$$\begin{aligned} & \text{B} \xrightarrow{7} \text{S} \xrightarrow{8} \text{M} \xrightarrow{13} \text{K} = 28 \\ & \text{B} \xrightarrow{7} \text{S} \xrightarrow{9} \text{O} \xrightarrow{11} \text{K} = 27 \\ & \text{B} \xrightarrow{7} \text{S} \xrightarrow{8} \text{M} \xrightarrow{5} \text{O} \xrightarrow{11} \text{K} = 31 \end{aligned}$$

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Dijkstra's algorithm

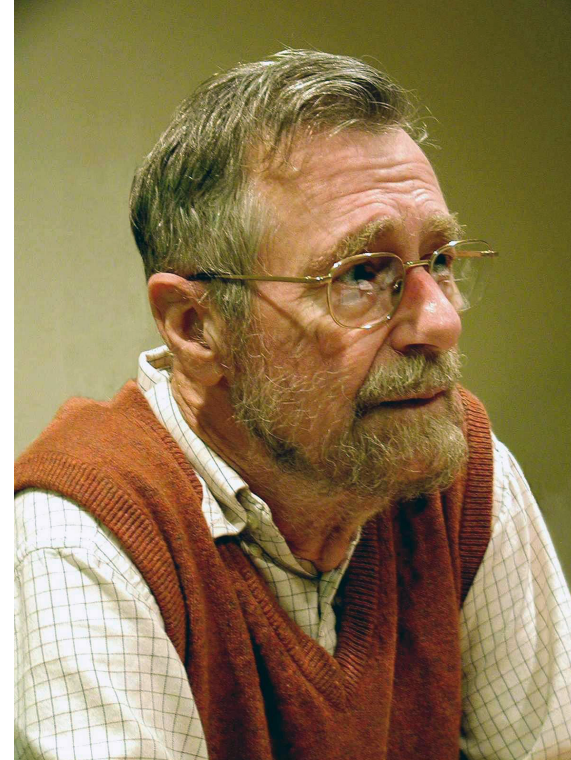
OK! Another person who had too much time on their hands and an over excitement about Mathematics.

Edsger Wybe Dijkstra

11 May 1930 – 6 August 2002)

A Dutch computer scientist, programmer, software engineer, systems scientist, science essayist,[9][10] and pioneer in computing science.[11] A theoretical physicist by training, he worked as a programmer at the Mathematisch Centrum (Amsterdam) from 1952 to 1962. A university professor for much of his life, Dijkstra held the Schlumberger Centennial Chair in Computer Sciences at the University of Texas at Austin from 1984 until his retirement in 1999. He was a professor of mathematics at the Eindhoven University of Technology (1962–1984) and a research fellow at the Burroughs Corporation (1973–1984). In 1972, he became the first person to win the Turing Award who was neither American nor British.

Extracted from Wikipedia



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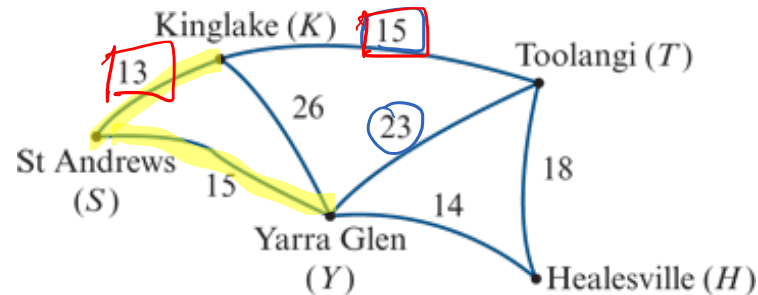
Dijkstra's algorithm

When there are many vertices and many edges, a systematic method, called an algorithm, can be used to find the shortest path.

Dijkstra's algorithm can be used to find the shortest path through a network, from a starting vertex to a destination vertex.

Let's work through this with an example.

The algorithm will be used to find the shortest path between St Andrews (S) and Toolangi (T).



$$\begin{array}{r} 23 \\ +15 \\ \hline 38 \end{array}$$

$$\begin{array}{r} +13 \\ = \\ +15 \end{array}$$

S - K - T

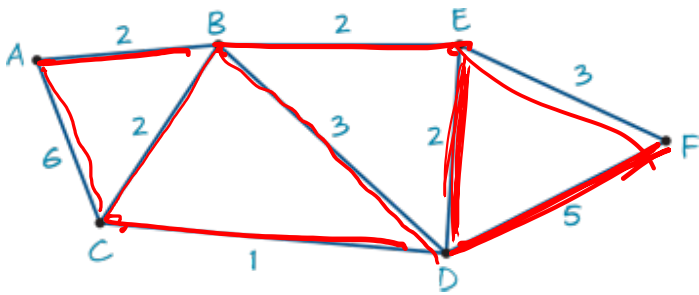
	K	Y	T	H
S	13	15	x	x
K	13	15	28	x
Y	13	18	28	29

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Another example of Dijkstra's algorithm

Find the shortest path from A to F in the weighted graph shown on the right



		↓ B	↓ C		↓ D		↓ E		↓ F
	A	2	6		X		X		X
+2	B	2	4		5		4		X
+4	C	2	4		5		4		X
→ +4	E	2	4		5		4		7
+8	D	2	4		5		4		7

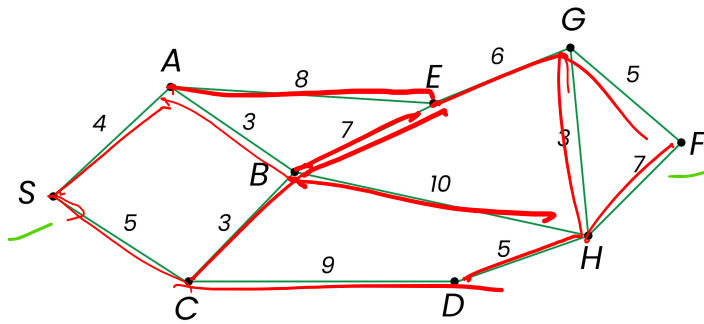
A - B - E - F

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One more example of Dijkstra's algorithm

Find the shortest path from S to F in the weighted graph shown on the right



S-A-E-G-F

		A	B	C	D	E	G	H	F
S		4	x	5	x	x	x	x	x
+4 A		4	7	5	x	12	x	x	x
+5 C		4	7	5	14	12	x	x	x
+12 E		4	7	5	14	12	18	x	x
+7 B		4	7	5	14	12	18	17	x
+14 D		4	7	5	14	12	18	17	x
+17 H		4	7	5	14	12	18	17	24
+18 G		4	7	5	14	12	18	17	23

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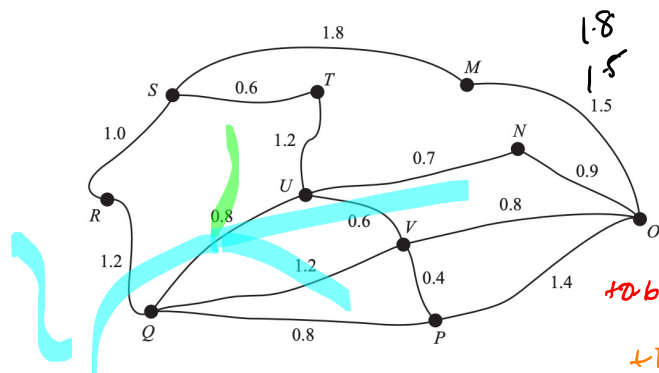
VCAA Question from 2020

Question 3 (4 marks)

A local fitness park has 10 exercise stations: M to V .

The edges on the graph below represent the tracks between the exercise stations.

The number on each edge represents the length, in kilometres, of each track.



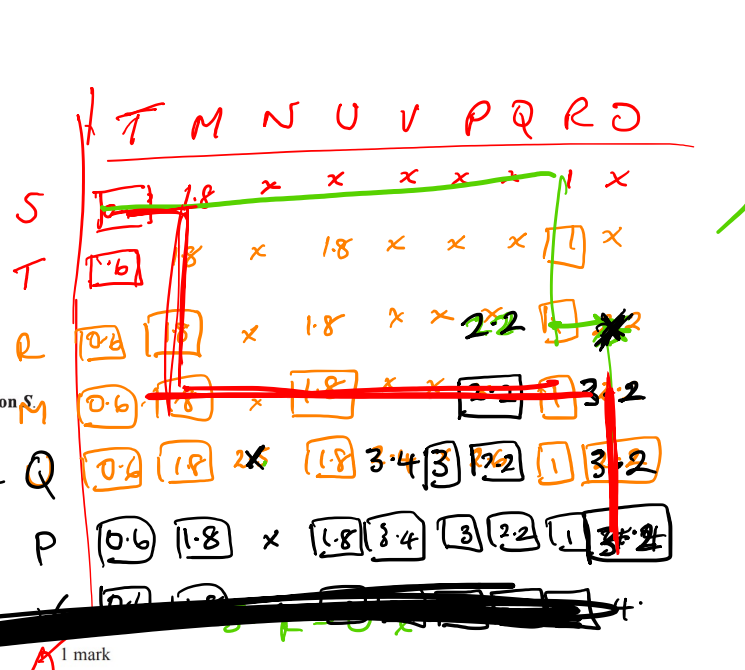
The Sunny Coast cricket coach designs three different training programs, **all starting at exercise station S**

Training program number	Training details
1	The team must run to exercise station O .
2	The team must run along all tracks just once.
3	The team must visit each exercise station and return to exercise station S .

- a. What is the shortest distance, in kilometres, covered in training program 1?

b. i. What mathematical term is used to describe training program 2?

- ii. At which exercise station would training program 2 finish?



$$\underline{\underline{S-M-D = 3-2}}$$

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