

# Trees and minimum connector problems



**Year 12  
Further Mathematics**

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"This is fantastic! Especially in these current circumstances. You just saved me from making this exact video for my students."

Youtube comment  
(from a current Teacher)

"Your british accent is so intriguing"

Youtube comment (Other YouTuber)

"Very helpful. I'm having trouble with mathematics during quarantine, I've found this channel off a tiktok one of your students has made and it's much easier now!"

Youtube comment  
(from current Year 12 student)

"Mate. You're bloody awesome. To be a maths teacher and to make this stuff.

You know... brilliant."

Youtube comment  
(from current Year 12 student)

"Thank you so much for your videos! Especially now that we have had to move to remote learning they have been a life saver!!"

Youtube comment  
(from current Year 12 student)

## Learning Objectives

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By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Further Mathematics course.

- Understand what a tree is
- Understand what spanning trees are
- Know how to find minimum spanning trees
- Understand what it means by connector problem

## Recap of past learning

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This is the last part of this section of the course and it's one of the more interesting! We are now going to look at something called Trees.

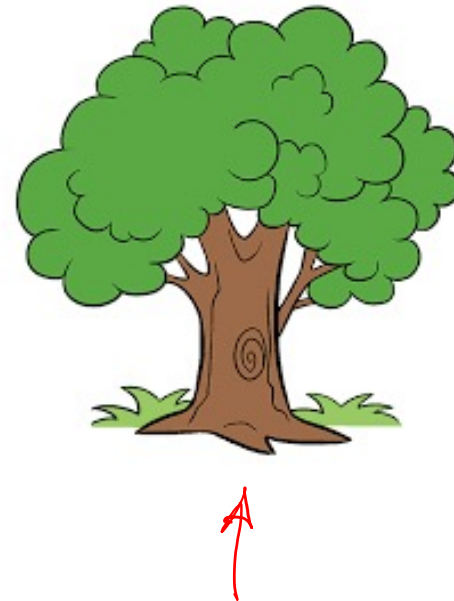
Probably not the type of tree shown on the right, but it's a good analogy for what we are going to be looking at.

**Most graphs seem to have loops or multiple edges or cycles.**

A tree has none of these. Which actually mimics the way a real tree grows.

**Remember:**

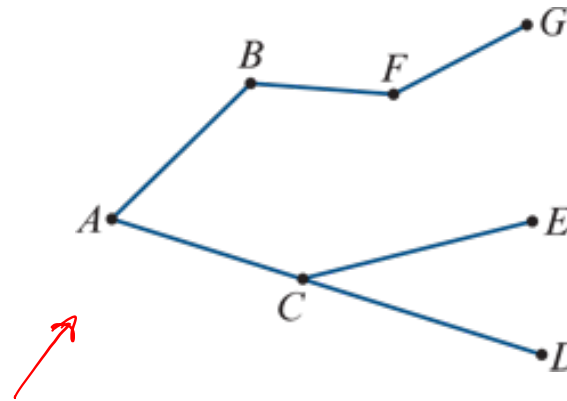
- A connected graph has every vertex connected to every other vertex, either directly or indirectly via other vertices.
- A subgraph is a part of a larger graph



## Example of a tree

Here is a wonderful example of a tree. Notice that it has no loops, multiple edges or cycles.

Vertices = 7  
edges = 6



Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

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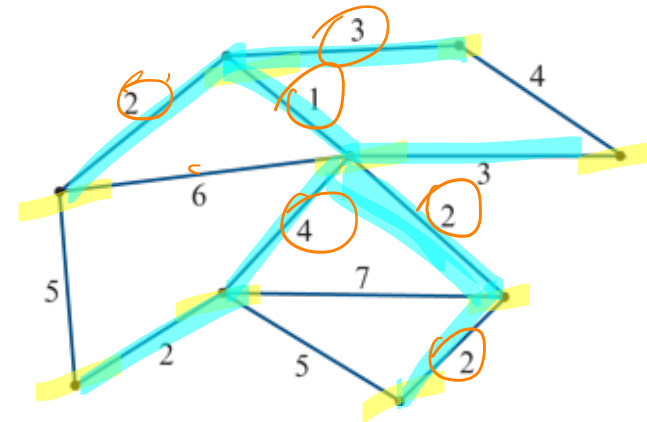
## Spanning tree

Every connected graph will have at least one subgraph that is a tree. A subgraph is a tree, and if that tree connects all of the vertices in the graph, then it is called a spanning tree.

When we add all the edges of the spanning tree we are finding its **weight**.

### Example:

Draw one spanning tree for the graph shown.  
Calculate the weight of the spanning tree.

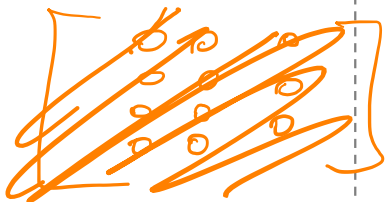


9 vertices

8 edges

18

$$2 + 3 + 1 + 4 + 2 + 2 + 2$$



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## Minimum spanning tree and Primm's Algorithm

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When we are looking for the **minimum spanning tree** we are looking to find the spanning tree with the smallest weight. We can do this using something called Primm's Algorithm.

There are four stages to the algorithm:

- Choose a starting vertex (any will do).
- Inspect the edges starting from the starting vertex and choose the one with the lowest weight. (If there are two edges that have the same weight, it does not matter which one you choose). The starting vertex, the edge and the vertex it connects to form the beginning of the minimum spanning tree.
- Now inspect all of the edges starting from both of the vertices you have in the tree so far. Choose the edge with the lowest weight, ignoring edges that would connect the tree back to itself. The vertices and edges you already have, plus the extra edge and vertex it connects form the minimum spanning tree so far.
- Keep repeating this process until all of the vertices are connected.



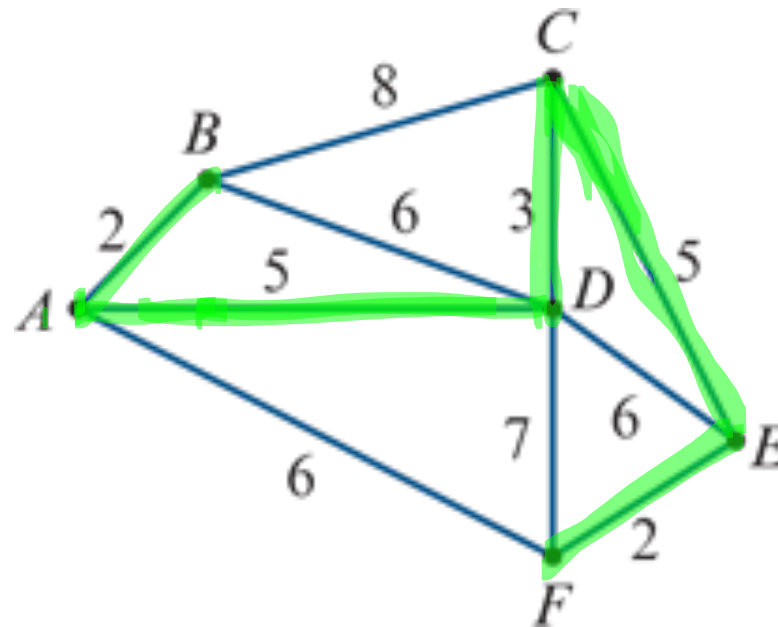
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## Example of Prim's Algorithm

Apply Prim's algorithm to find the minimum spanning tree for the graph shown on the right. Write down the total weight of the minimum spanning tree.

- Choose a starting vertex (any will do).
- Inspect the edges starting from the starting vertex and choose the one with the lowest weight. (If there are two edges that have the same weight, it does not matter which one you choose). The starting vertex, the edge and the vertex it connects to form the beginning of the minimum spanning tree.
- Now inspect all of the edges starting from both of the vertices you have in the tree so far. Choose the edge with the lowest weight, ignoring edges that would connect the tree back to itself. The vertices and edges you already have, plus the extra edge and vertex it connects form the minimum spanning tree so far.
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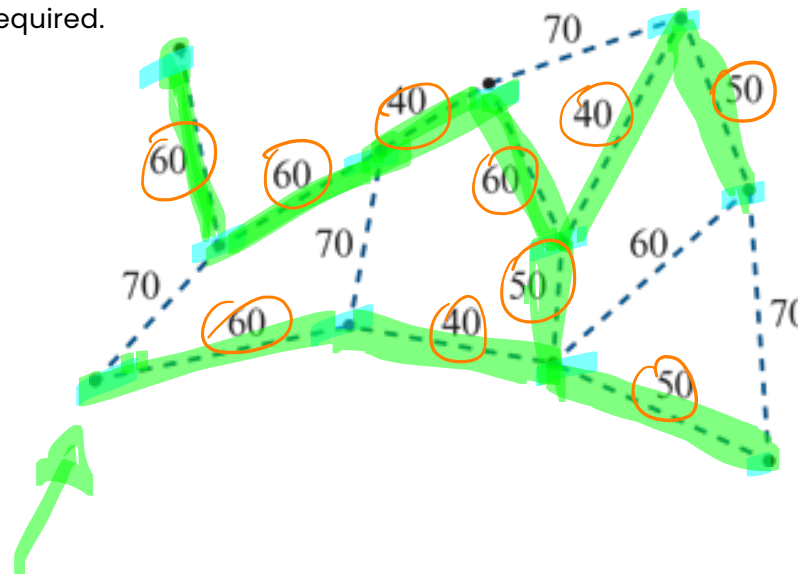
## Connector problems

A connector problem uses minimum spanning trees with some relationship to money. When finding the minimum it's also helping find the cheapest way to do something.

Example (**from VCAA**)

At a showgrounds, 11 locations require access to water. These locations are represented by vertices on the network diagram shown. The dashed lines on the network diagram represent possible water pipe connections between adjacent locations. The numbers on the dashed lines show the minimum length of pipe, in metres, required to connect these locations.

- On the diagram, show where these water pipes will be placed.
- Calculate the total length, in metres, of water pipe that is required.



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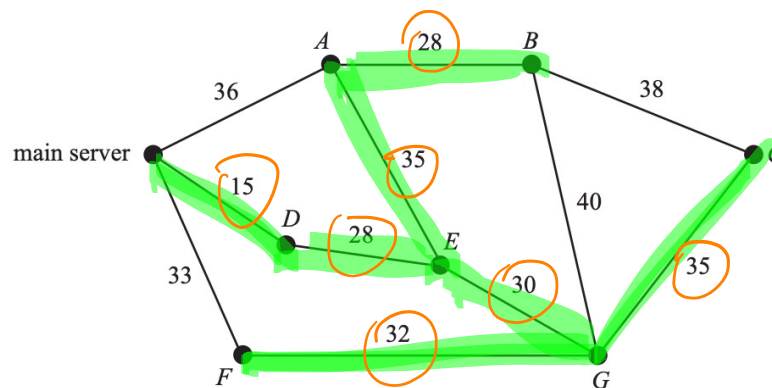
## VCAA Question

Here is an example of a questions which appeared in the VCAA Further Maths 2019 Paper 1

Q7

### Question 5

The following diagram shows the distances, in metres, along a series of cables connecting a main server to seven points,  $A$  to  $G$ , in a computer network.



The minimum length of cable, in metres, required to ensure that each of the seven points is connected to the main server directly or via another point is

- A. 175
- B. 203
- C. 208
- D. 221
- E. 236

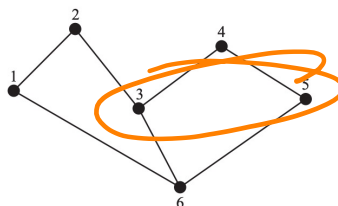
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## VCAA Question

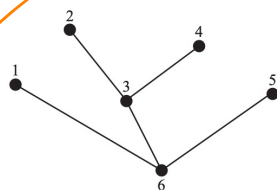
Here is an example of a VCAA question which appeared on the 2020 Paper 1 Exam

Question 3

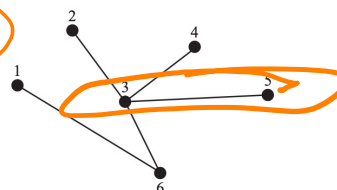


Which one of the following is **not** a spanning tree for the network above?

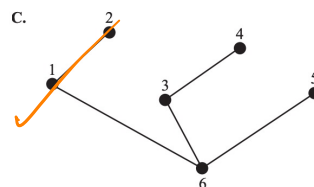
A.



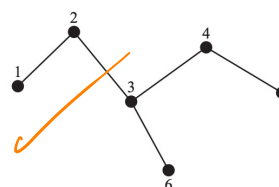
B.



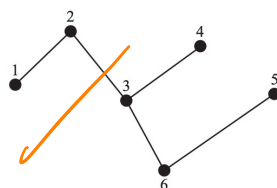
C.



D.



E.



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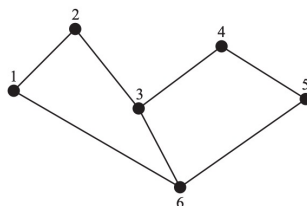
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## VCAA Question

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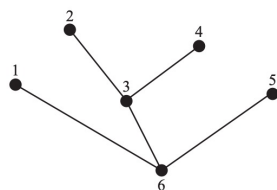
~~14 E~~  
14 Review.

Question 3

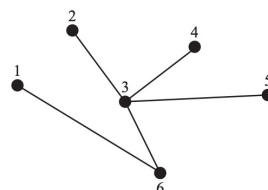


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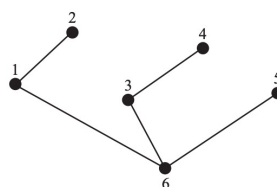
A.



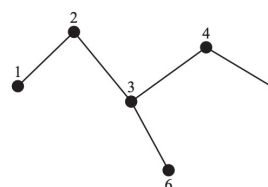
B.



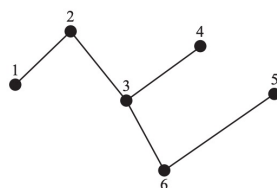
C.



D.



E.



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## VCAA Questions

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