The sine rule

Sunday, 20 October 2019 12:36 pm

By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know that we have a way to find missing sides and angles for non-right angled triangles
- Know that we can use the Sine Rule to find the missing sides and angles for non-right angled triangle when we have two side lengths and one opposite angle
- Know how to use the sine rule.

RECAP

The Maths just keeps coming. Having now looked at the ideas of sine, cosine and tangent for right angled triangles, we need to find ways to find angles for non-right angled triangles. This is going to be new to most of the students following this course. The good news is … it's one formula and we can use our calculators to help to use problems using the Sine Rule.

The Sine Rule

Let's start of with seeing what the Sine Rule looks like.

WARNING! It's going to look disgusting … probably because you are going to look at the fractions first!

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Firstly: Similar Fractions

In Years 7 and 8 we looked at the following ideas:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

$$\frac{1}{2} = \frac{2}{4}$$

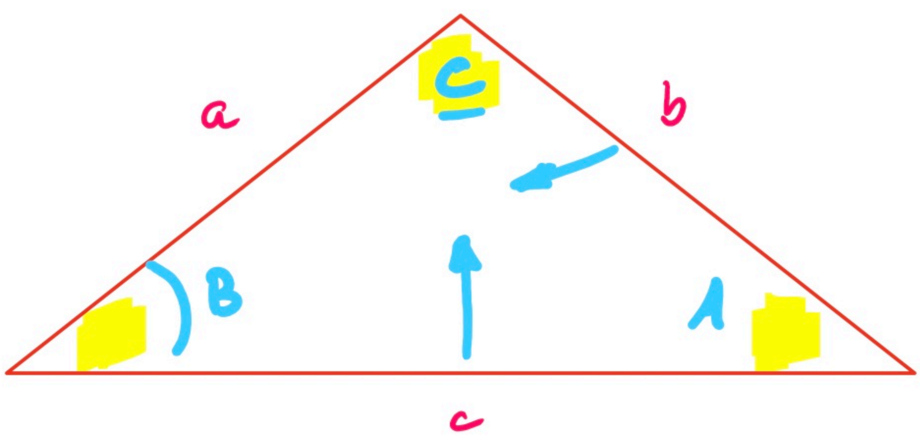
$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{3}{6} = \frac{4}{8}$$

This is the same idea as above.
We are just using ratios to help us find missing sides and angles.

The names of the sides are changing again!

We have already got used to the idea that for Pythagoras we have the sides labelled as one thing.
Then we moved to Trigonometry where the sides were labelled a different thing.
We are now going to do it again!



The thing to remember is the difference between capital and non-capital letters AND that they are opposite each other!

CAPITAL LETTERS stand for angles.
Non capital letters stand for side lengths.

Note: When using the formula we ONLY ever use two parts of it

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

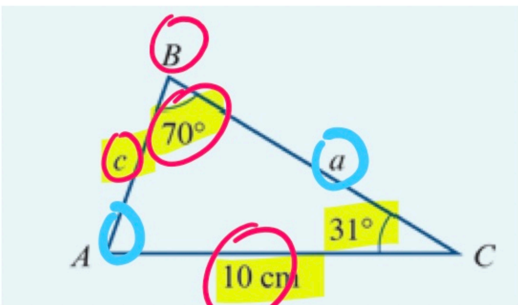
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Example 1
Find the length of AB.



$$\cancel{\frac{a}{\sin A}} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

F

$$\frac{10}{\sin 70} = \frac{c}{\sin 31}$$

S

sin31

$$\frac{10}{\sin 70} = \frac{c}{\sin 31} \quad \times \sin 31$$

$$\frac{10}{\sin 70} \times \sin 31 = c$$

$c = 5.48$ cm

Example 2

Find possible values for the magnitude of angle XZY in the triangle XYZ, given that $Y = 25°$, $y = 5\ cm$ and $z = 6\ cm$.

In the case above … there are actually two triangles which fit this.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \cancel{\frac{c}{\sin C}}$$

$$\frac{5}{\sin 25} = \frac{6}{\sin\theta}$$

$0 \le x \le 180$

$$\therefore \theta = 30.5°, \quad \theta = 149.5°$$

$$\frac{5}{\sin 25} = \frac{6}{\sin\theta}$$

$$\frac{\sin\theta}{\sin 25} = \frac{6}{5}$$

$$\sin\theta = \frac{6}{5} \times \sin 25$$

$$\sin\theta = 0.507\ldots$$

$$\theta = \sin^{-1}(0.507\ldots)$$

$$\theta = \underline{30.5°}$$

$$\theta = 180 - 30.5° = \underline{149.5°}$$