★ By the end of the lesson I would hope that you have an understanding (and be able to apply to questions) the following concepts:

- Know that we have a way to find missing sides and angles for non-right angled triangles
- Know that we can use the Cosine Rule to find the missing sides and angles for non-right angled triangle when we have two side lengths and one included angle
- Know how to use the Cosine rule.

RECAP

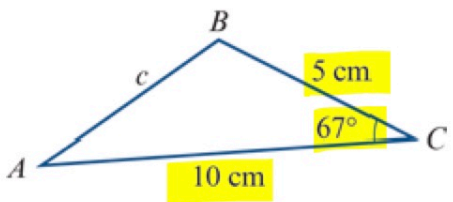
In the previous two lessons we have recapped the work covered with Trigonometry (which was covered in Years 9 and 10) and then moved onto looking at triangles which don't have right angles. There are a number of new rules we can use to help find the missing sides and angles of non right-angled triangles One of them you have already met with the Sine Rule.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This works when:

- we have two sides and one opposite angle
- Two angles and one opposite side

What would happen with the following example?



We can't seem to use the Sine Rule for this.

Welcome to the Cosine Rule

Knowing when to use the Cosine Rule

Basically, we know to use the cosine rule when there is love in the air.

OK. Love to me is the feeling being **included**.

When I'm included I'm in the middle …

Hence, we use the Cosine Rule for the following cases

- when we have two sides and one included angle.
- When we have three sides and need to find an angle

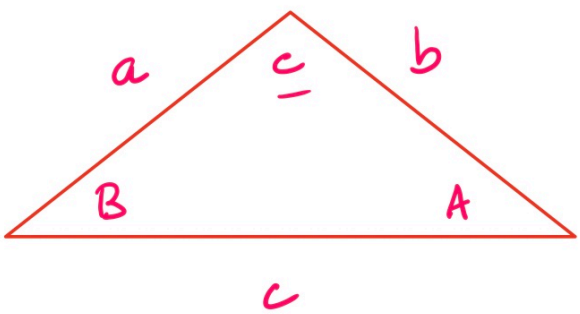
Unlike the Sine Rule there are two formulae to remember (one is just the transposition of the other):

$$a^2 = b^2 + c^2 - 2 \times b \times c \times \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$

What do the letters stand for?

The same as they did with the Sine Rule!

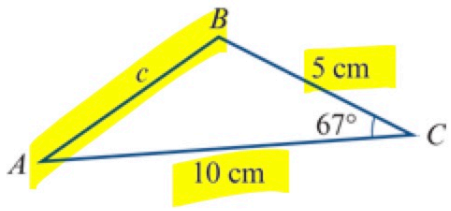


The best way to know how to use the Cosine Rule is with some examples

Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

For triangle ABC, find the length of AB in centimetres, correct to two decimal places.



F $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

S: $a^2 = 5^2 + 10^2 - 2.5.10 \cdot \cos(67)^\circ$

$a^2 = 25 + 100 - 100 \cdot \cos 67^\circ$

$a^2 = 125 - 100 \cos 67^\circ$

$a^2 = 85.9268\ldots$

$a = \sqrt{85.9268\ldots}$

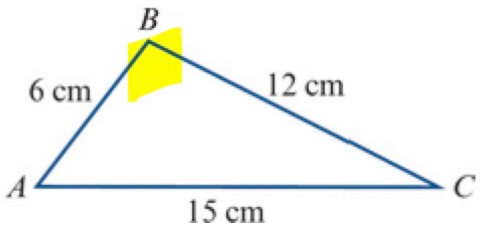
$a = 9.27 \text{ cm}$

Examples

The following examples are used, with permission, from the Cambridge Further Mathematics Units 3 and 4 textbook.

Find the magnitude of angle ABC for triangle ABC.

Give your answer correct to two decimal points.



$\angle ABC$

F: $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

$\cos B = \dfrac{6^2+12^2-15^2}{2.6.12}$

$\cos B = -0.3125$

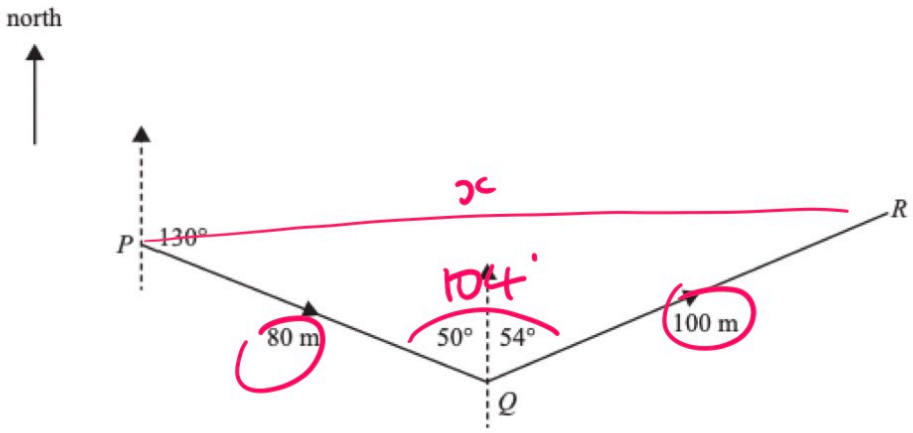
$B = \cos^{-1}(-0.3125)$

$= 108.21^\circ$

VCAA Exam Question on this concept
2016 Paper 2

Question 4 (3 marks)

During a game of golf, Salena hits a ball twice, from P to Q and then from Q to R.
The path of the ball after each hit is shown in the diagram below.



After Salena's first hit, the ball travelled 80 m on a bearing of 130° from point P to point Q.
After Salena's second hit, the ball travelled 100 m on a bearing of 054° from point Q to point R.

a. Another ball is hit and travels directly from P to R.

Use the cosine rule to find the distance travelled by this ball.
Round your answer to the nearest metre. 2 marks

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$x^2 = 80^2 + 100^2 - 2 \cdot 80 \cdot 100 \cdot \cos 104°$

$x^2 = 20\,270.75033$

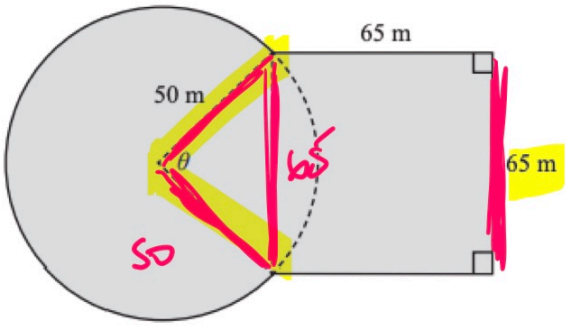
$x = 142 \text{ m}$

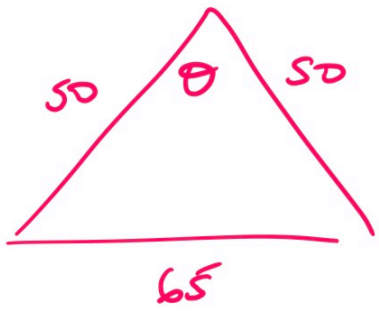
VCAA Exam Question on this concept
2017 Paper 2

Question 3 (4 marks)

The hostel buildings are arranged around a grassed area.

The grassed area is shown shaded in the diagram below.





The grassed area is made up of a square overlapping a circle.

The square has side lengths of 65 m.

The circle has a radius of 50 m.

An angle, θ, is also shown on the diagram.

a. Use the cosine rule to show that the angle θ, correct to the nearest degree, is equal to 81°. 1 mark

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

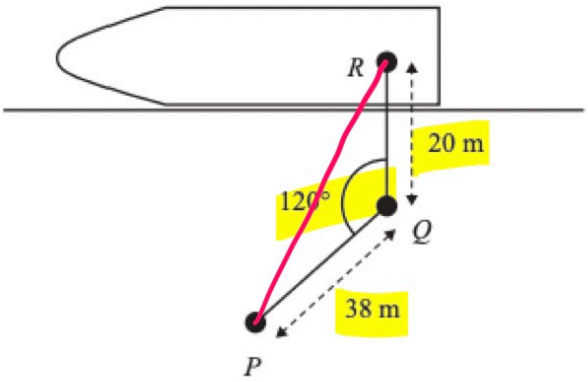
$$\cos A = \frac{50^2 + 50^2 - 65^2}{2 \cdot 50 \cdot 50}$$

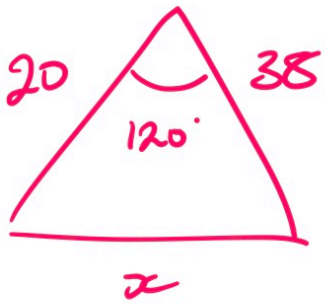
$$\cos A = 0.155$$

$$A = 81°$$

VCAA Exam Question on this concept
2019 Paper 2

b. The diagram below shows a cargo ship next to a port. The base of a crane is shown at point Q.





The base of the crane (Q) is 20 m from a shipping container at point R. The shipping container will be moved to point P, 38 m from Q. The crane rotates 120° as it moves the shipping container anticlockwise from R to P.

What is the distance RP, in metres?
Round your answer to the nearest metre. 1 mark

$x^2 = b^2 + c^2 - 2bc.\cos A$

$x^2 = 20^2 + 38^2 - 2.20.38.\cos 120°$

$x^2 = 2604$

$x = 51m$

$RP = 51m$