



Linear models

**Year 12 Further Maths
Units 3 and 4**

www.maffsguru.com



This is where
Darren goes



Maths made ...
Easy
Engaging
Educational
Entertaining

Sign up for free!

Videos come with downloadable notes at
www.maffsguru.com

Learning Objectives

By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand how to apply the learnings from previous lessons to linear model questions

This is where
Darren goes

Recap

Much of what we do in life can be modelled by straight line relationships.

For example, if we travel at a constant speed, we can plot a linear relationship between time and distance.

When I get into a cab, if we were dealing with a per Km pricing model, we can relate the distance travelled to the cost of the cab journey.

We can model the cost of hiring a car.

This is where
Darren goes

Language, language and more language

We need to remember that Barry loves tricking us!

When we deal with money or costs we need to think of two terms; fixed costs and variable costs.

Fixed costs are those which don't change regardless of how much I use something.

A cab has a fixed cost when then get into the cab.

Hiring a car for a day has a fixed cost which has nothing to do with the distance you travel.

This is where
Darren goes

Fixed costs are the intercept!

Fixed costs are, generally, the question telling you the value of a y-axis intercept.

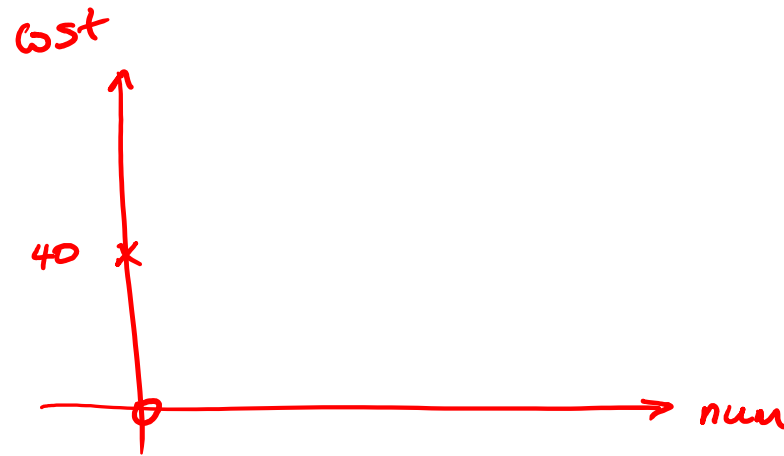
For example:

Austcom's rates for local calls from private telephones consist of a quarterly rental fee of \$40 plus 25c for every call. Construct a linear rule that describes the quarterly telephone bill and sketch the graph.

Here, the fixed cost is called a "Rental fee"

$$m =$$

$$c =$$



This is where
Darren goes

Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

What about the gradient?

The gradient can be found from the other information given in the question.

For example:

Austcom's rates for local calls from private telephones consist of a quarterly rental fee of \$40 plus 25c for every call. Construct a linear rule that describes the quarterly telephone bill and sketch the graph.

Here, the gradient would be 0.25 (**not 25!**).

$\$40$
 $\$0.25$ ← m

This is where
Darren goes

Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

Decimals to fractions

In the example:

Austcom's rates for local calls from private telephones consist of a quarterly rental fee of \$40 plus 25c for every call. Construct a linear rule that describes the quarterly telephone bill and sketch the graph.

We found the gradient to be 0.25.

But, to be able to plot this on a graph, we need to know what this is as a fraction.

There are a number of fractions we really should learn.

$$0.5 = \frac{1}{2}$$

$$0.25 = \frac{1}{4}$$

$$0.125 = \frac{1}{8}$$

$$0.1 = \frac{1}{10}$$

$$c = 40$$

$$m = 0.25$$

$$= \frac{1}{4} \quad \leftarrow \begin{array}{l} \text{rise} \\ \text{run} \end{array}$$

This is where
Darren goes

Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

Completing the question

Let's now construct the rule and sketch the graph:

Austcom's rates for local calls from private telephones consist of a quarterly rental fee of \$40 plus 25c for every call. Construct a linear rule that describes the quarterly telephone bill and sketch the graph.

Note: Assume that the minimum number of calls is 0. Plot the point which corresponds to the number of calls being 200.

$$m = \frac{1}{4}$$
$$c = 40$$
$$y = mx + c$$
$$\text{Cost} = \frac{1}{4} \times \text{Num} + 40$$
$$\text{Cost} = \frac{1}{4} \times 200 + 40$$
$$= 50 + 40$$
$$= \$90$$



papergraph.com © 3.0 by-nc-sa Peter Milne

This is where
Darren goes

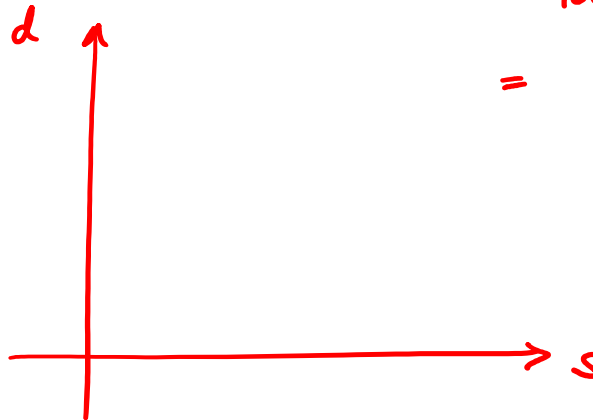
Examples have been extracted, with permission, from
the Cambridge Further Mathematics Units 3 and 4 Textbook

A more complex example: Being given two coordinates

The tyres on a racing car had lost 3 mm of tread after completing 250 km of a race and 4 mm of tread after completing 1000 km. Assuming that the loss of tread was proportional to the distance covered, find the total loss of tread, d mm, after s km from the start of the race. What would be the tread loss by the end of a 2000 km race? Give your answer correct to one decimal place.

In the above example, they have given us two coordinates.

Let's find the relationship between d and s (which is assumed to be linear relationship).



$$\begin{aligned}
 & (250, 3) \qquad \downarrow \qquad (1000, 4) \\
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{4 - 3}{1000 - 250} \\
 &= \frac{1}{750} \\
 y - 4 &= \frac{1}{750} (x - 1000) \\
 y &= \frac{1}{750} (x - 1000) + 4 \\
 &= \frac{1}{750} x - \frac{1000}{750} + 4 \\
 y &= \frac{1}{750} x + 2.67 \\
 d &= \frac{1}{750} \cdot s + 2.67
 \end{aligned}$$

This is where
Darren goes

Examples have been extracted, with permission, from
the Cambridge Further Mathematics Units 3 and 4 Textbook

A more complex example: Being given two coordinates

The tyres on a racing car had lost 3 mm of tread after completing 250 km of a race and 4 mm of tread after completing 1000 km. Assuming that the loss of tread was proportional to the distance covered, find the total loss of tread, d mm, after s km from the start of the race. What would be the tread loss by the end of a 2000 km race? Give your answer correct to one decimal place.

Now we have a relationship, we can use it to find the answer to the question.

$$d = \frac{1}{750}s + 2.67$$

$$d = \frac{1}{750} \cdot s + 2.67$$

$$\begin{aligned} d &= \frac{1}{750} \times 2000 + 2.67 \\ &= \underline{5.3 \text{ mm}} \end{aligned}$$

This is where
Darren goes

The screenshot shows a scientific calculator interface with the following elements:

- Top Bar:** Edit Action Interactive
- Input Field:** $\frac{1}{750} * 2000 + 2.67$
- Result Field:** 5.336666667 (underlined in red)
- Function Grid:**

Math1	Line	$\frac{\square}{\square}$	$\sqrt{\square}$	π	\rightarrow
Math2	\square^{\square}	e^{\square}	ln	$\log_{\square}(\square)$	$\sqrt[\square]{\square}$
Math3	$ \square $	x^2	x^{-1}	$\log_{10}(\square)$	solve(
Trig	$\square \square$	toDMS	{ \square }	{ }	()
Var	sin	cos	tan	$^{\circ}$	r
abc				ans	EXE
- Bottom Bar:** Alg, Decimal, Real, Rad, $\frac{\square}{\square}$

Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

Thanks for watching

All videos are available to view at www.maffsguru.com
Lesson notes can be downloaded too

Please visit www.youtube.com/maffsguru and subscribe