Exploring and travelling

Year 12
Further Mathematics

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"This is fantastic! Especially in these current circumstances. You just saved me from making this exact video for my students."
Youtube comment
(from a current Teacher)

"Your british accent is so intriguing"
Youtube comment (Other YouTuber)

"Very helpful. I'm having trouble with mathematics during quarantine, I've found this channel off a tiktok one of your students has made and it's much easier now!"
Youtube comment
(from current Year 12 student)

"Mate. You're bloody awesome. To be a maths teacher and to make this stuff. You know... brilliant."
Youtube comment
(from current Year 12 student)

"Thank you so much for your videos! Especially now that we have had to move to remote learning they have been a life saver!!"
Youtube comment
(from current Year 12 student)

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Further Mathematics course.

- Understand what it means to travel
- Know the difference between the following:
  - Walk
  - Trail
  - Path
  - Circuit
  - Cycle
- Know and understand what an Eulerian trail is
- Know and understand what an Eulerian circuit is
- Know and understand what a Hamiltonian path is
- Know and understand what a Hamiltonian cycle is

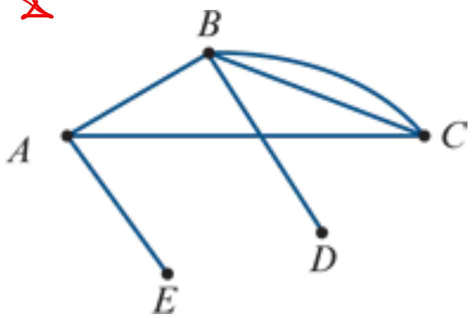
Recap of past learning

We are moving apace through this section of the course and now we have to stop and learn some more language and, to be honest, I've not been able to find an entertaining way to teach this (and hence learn it). So, perhaps you can let me know how you learned it!

We have now looked at graphs (and the language used to describe it). We've looked at the different types of graphs. Euler's rule to relate faces, edges and vertices. We've looked at how to use matrices to describe a graph.

It's now time to walk around these things!

	A	B	C	D	E
A	0	1	1	0	1
B	1	0	2	1	0
C	1	2	0	0	0
D	0	1	0	0	0
E	1	0	0	0	0



Why do we draw graphs?

Good question!

There are practical applications to this stuff. Logistics operators use it to find the shortest distance to move around a network for example. This means that goods are delivered quickly and, generally, with a reduced cost.

Moving around graphs can be described as travelling.

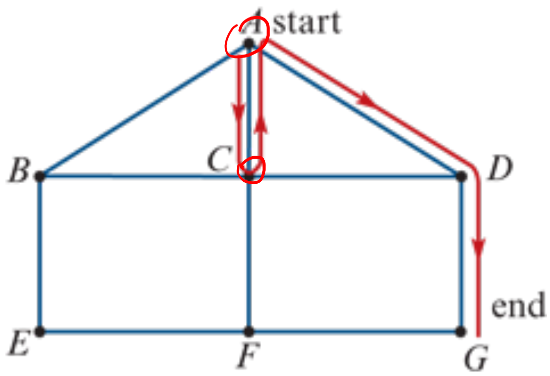
Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

Going for a 'walk'

Don't leave me yet! We can walk together later ...

Actually going for a walk means we go through a sequence of edges linking vertices in a graph.

A walk starts at one vertex and follows any route to finish at another vertex.



A → C → A → D → G

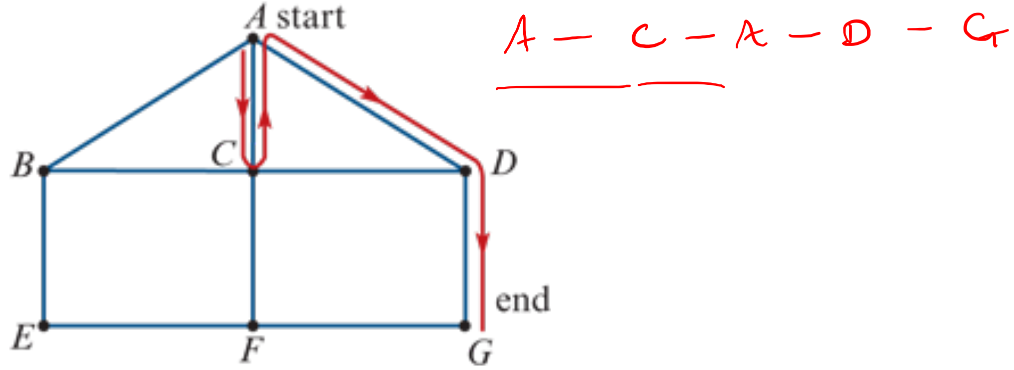
Examples have been extracted, with permission, from the Cambridge Further Mathematics Units 3 and 4 Textbook

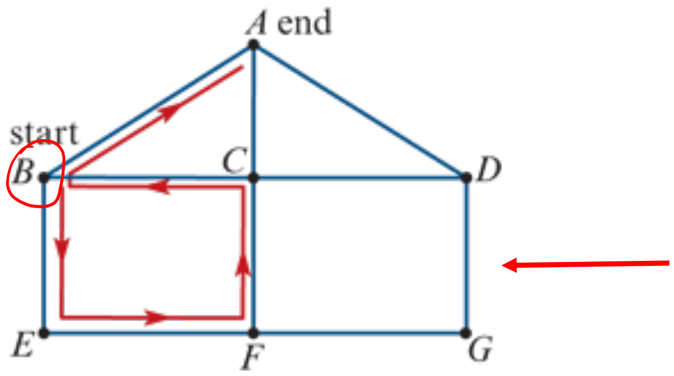
Following a 'trail'

A trail is a walk with **no repeated edges**.

Hence the first diagram would not be a trail as we repeated edge AC.

Note: We can repeat vertices though!





This is a trail

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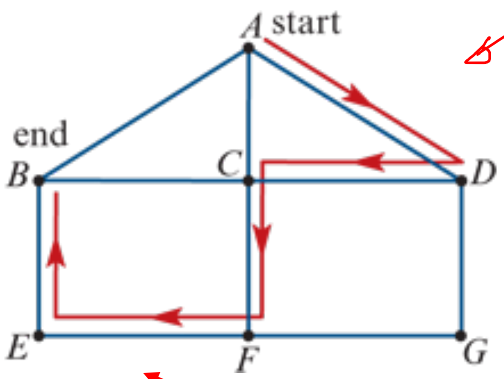
Don’t stray far from the ‘path’

A path is a walk with no repeated edges and no repeated vertices.



A – D – C – F – E – B

This is a path

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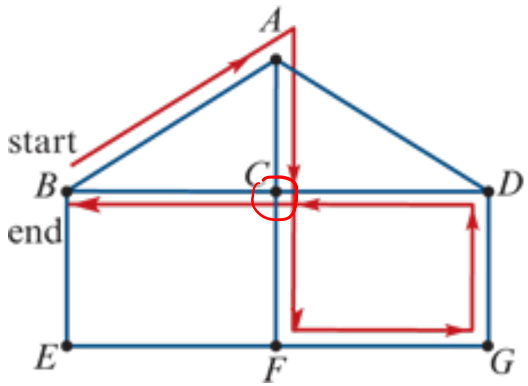
Doing the dinner party 'circuit'

A circuit is a trail (no repeated edges) that starts and ends at the same vertex. Circuits are also called closed trails.



This is a circuit

B - A - C - F - G - D - C - B

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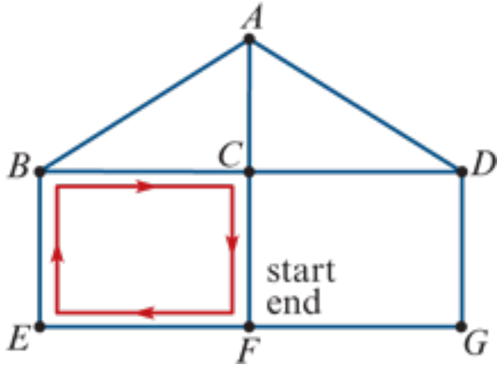
Going for a ride on my 'cycle'

A cycle is a path (no repeated edges, no repeated vertices) that starts and ends at the same vertex. The start and end vertex is an exception to repeated vertices. Cycles are also called closed paths.



F - E - B - C

This is a cycle

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Back to Euler again …

OK … so this is Euler … can we now move on?

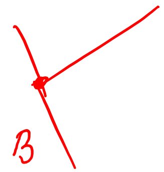
Leonhard Euler
15 April 1707 – 18 September 1783
A Swiss mathematician, physicist, astronomer, geographer, logician and engineer who made important and influential discoveries in many branches of mathematics, such as infinitesimal calculus and graph theory, while also making pioneering contributions to several branches such as topology and analytic number theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function.[4] He is also known for his work in mechanics, fluid dynamics, optics, astronomy and music theory.[5]

Extracted from Wikipedia

Eulerian trails and circuits

Trails and circuits that follow every edge, without duplicating any edge, of a graph are called **eulerian trails** and **eulerian** circuits.

They can be identified using the following helpful points:



Eulerian trial:

- An eulerian trail follows every edge of a graph.
- An eulerian trail will exist if the graph:
  - is connected
  - has exactly two vertices that have an odd degree.

Eulerian circuit:

An eulerian circuit is an eulerian trail (follows every edge) that starts and ends at the same vertex.

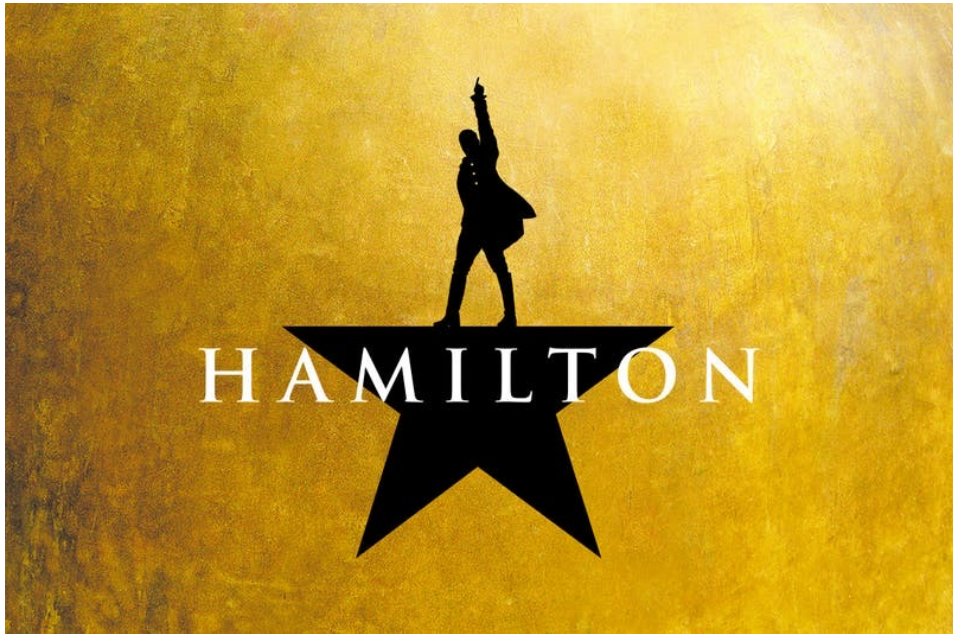
An eulerian circuit will exist if the graph:
- is connected
- has vertices that all have an even degree.

An eulerian circuit can begin at any of the vertices.

Who is Hamilton?

After a quick google search I came up with about 1,000,000 pictures and photos for the musical Hamilton, but I don't think he was the person who created the ideas behind paths and cycles.

Did they even have cycles then?



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Sir William Rowan Hamilton

Sir William Rowan Hamilton MRIA
(3 August 1805 – 2 September 1865)
An Irish mathematician, Andrews Professor of Astronomy at Trinity College Dublin, and Royal Astronomer of Ireland. He worked in both pure mathematics and mathematics for physics. He made important contributions to optics, classical mechanics and algebra. Although Hamilton was not a physicist–he regarded himself as a pure mathematician–his work was of major importance to physics, particularly his reformulation of Newtonian mechanics, now called Hamiltonian mechanics. This work has proven central to the modern study of classical field theories such as electromagnetism, and to the development of quantum mechanics. In pure mathematics, he is best known as the inventor of quaternions.

Extracted from Wikipedia

Hamiltonian paths and cycles

Paths and cycles that pass through every vertex of a graph only once are called hamiltonian paths and hamiltonian cycles, named after the mathematician William Rowan Hamilton.

Hamiltonian paths
A hamiltonian path visits every vertex of a graph.

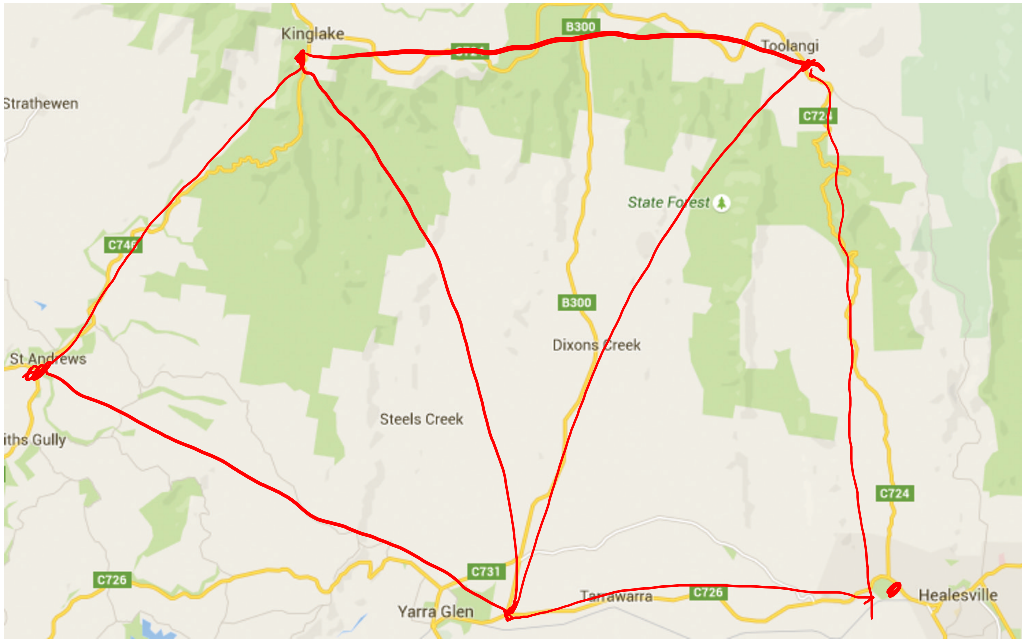
Hamiltonian cycles
A hamiltonian cycle is a hamiltonian path (every vertex) that starts and ends at the same vertex.

Note: Inspection is the only way to identify Hamilton paths and cycles.
Remember: Eulerian trails and circuits do not repeat edges. Hamiltonian paths and cycles do not repeat vertices.

Example

A map showing the towns of St Andrews, Kinglake, Yarra Glen, Toolangi and Healesville is shown below.

- **Draw a graph with a vertex representing each of these towns and edges representing the direct road connections between the towns.**
- Explain why an eulerian trail, but not an eulerian circuit, is possible through this graph.
- Write down an eulerian trail that begins at Toolangi.
- Write down a hamiltonian cycle that begins at Healesville.



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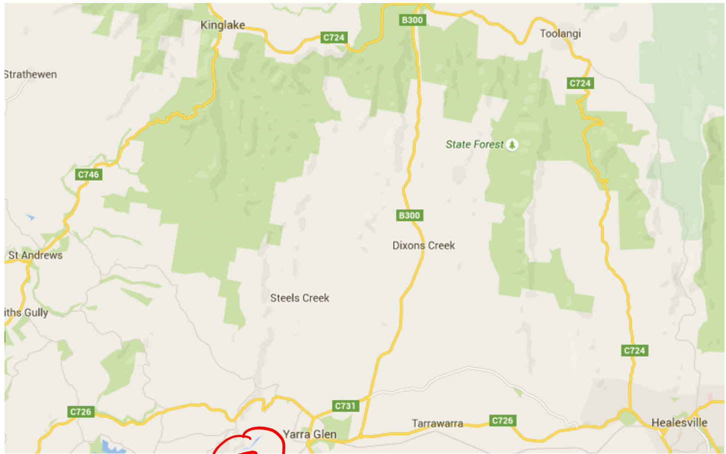

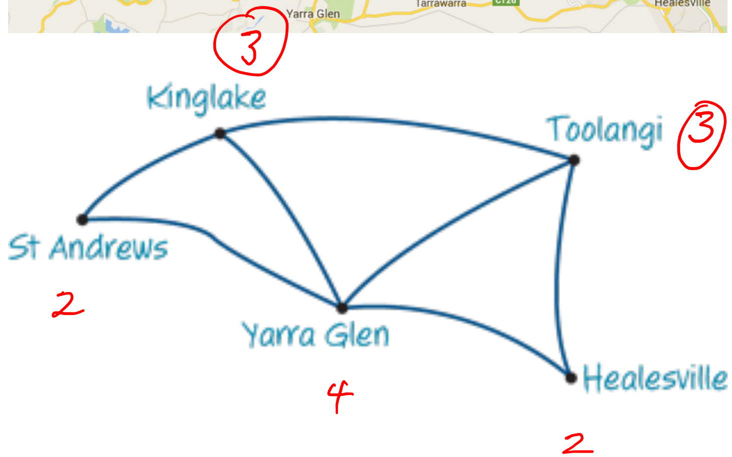

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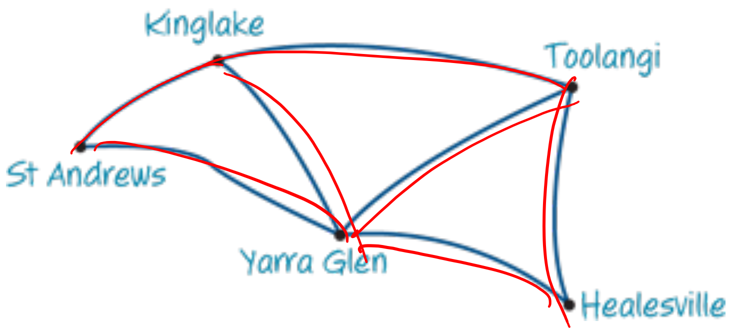

T – H – Y – T – K – Y – S – K

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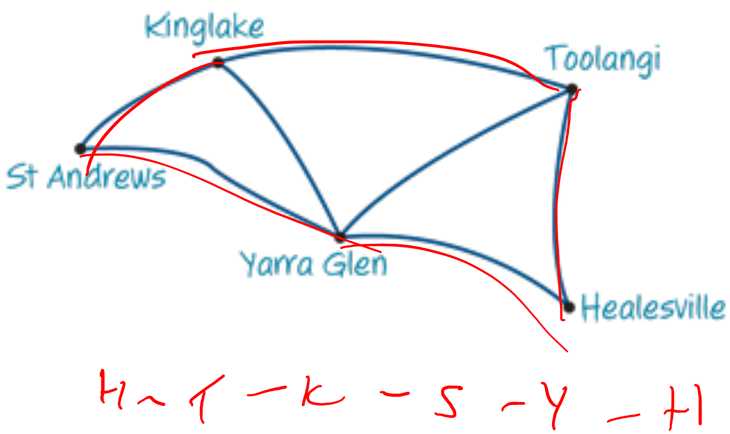

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Year 9

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