Differentiating x^n where n is a negative integer (9C)

Thursday, 8 March 2018 6:34 pm

Work to be completed by the end of teaching:

Negative integer powers	9C	1,5,6,7,9,11,12

RECAP:

We know, from the previous chapter, that the shortcut to differentiate is to:

- Multiply the coefficient by the power and then
- Reduce the power by 1

$y = 0x^{6}$
$y' = 6x^{5}$

This is true for negative powers also.

The trick with negative powers is to know how they can trick you!

Examples with basic questions:

$y = 6x^4$
$y' = 24x^3$

$y = x^3$
$y = \frac{1}{x^{-3}}$

$y = \frac{1}{x^2}$

$y = 0x^{-2}$

$y' = -2x^{-3}$

$y = \frac{-2}{x^3}$

$y' = -2 \times x^{-3}$
$= -2 \times \frac{1}{x^3}$
$= \frac{-2}{x^3}$

$y = 3x^{-2} + 6x^{-1} + 3x^2$
$y' = -6x^{-3} - 6x^{-2} + 6x$
$y' = -\frac{6}{x^3} - \frac{6}{x^2} + 6x$

Negative Powers: The Tricks

Trick 1: Division

$$f(x) = \frac{x^2 + 2x - 3}{x^3}$$

This can be done in one of two ways

$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4}$
$= \frac{2+1}{4}$
$= \frac{3}{4}$

Dividing all the numerator by the denominator to gain 3 separate terms

$$f(x) = x^{-1} + 2x^{-2} - 3x^{-3}$$

$f(x) = \frac{\cancel{x^2}}{x^{\cancel{3}}} + \frac{2\cancel{x}}{x^{\cancel{3}\,2}} - \frac{3}{x^3}$
$= \frac{1}{x} + \frac{2}{x^2} - \frac{3}{x^3}$
$= x^{-1} + 2x^{-2} - 3x^{-3}$

$x^{-3} \times x^2$

Turning the equation into the following form:

$$f(x) = (x^2 + 2x - 3)x^{-3}$$

$= x^{-3}(x^2 + 2x - 3)$
$= x^{-1} + 2x^{-2} - 3x^{-3}$
$f'(x) = -1x^{-2} - 4x^{-3} + 9x^{-4}$
$= -\frac{1}{x^2} - \frac{4}{x^3} + \frac{9}{x^4}$

Spoiler alert: Later on you will come across a method to do this called the **Quotient Rule**

Function Notation ...

With the function shown below ... we need to ensure that we understand the there will now be values of x for which this function is NOT defined.

$$f(x) = \frac{x^2 + 2x - 3}{x^3}$$

Hence we would need to write this function, if asked, as:

$f: \mathbb{R}\setminus\{0\} \Rightarrow \mathbb{R},\ f(x) = x^2 + 2x - 3$

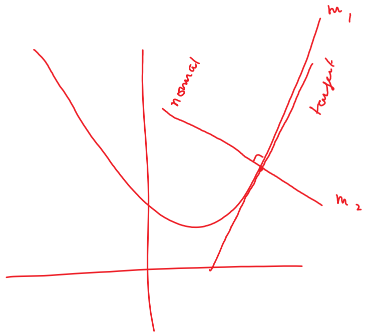
x^3

Applications of negative powers

Remember, the whole point of differentiation is to find the gradient of a tangent to a point.
Once we know the gradient of the tangent to a point, we can then proceed to find:

- the equation of the tangent, or
- the equation of the normal, and
- any points of intersection the tangent might have with the rest of the curve

$y = mx + c$



Example:

Find the x coordinates of the points on the curve $y = \dfrac{x^2 - 1}{x}$ at which the gradient of the curve is 5.

$$y = \frac{x^2 - 1}{x}$$

$$y = \frac{x^{\cancel{2}}}{\cancel{x}} - \frac{1}{x}$$

$$y = x - \frac{1}{x}$$

$$y = x - x^{-1}$$

$$y' = 1 + x^{-2}$$

$$y' = 1 + \frac{1}{x^2}$$

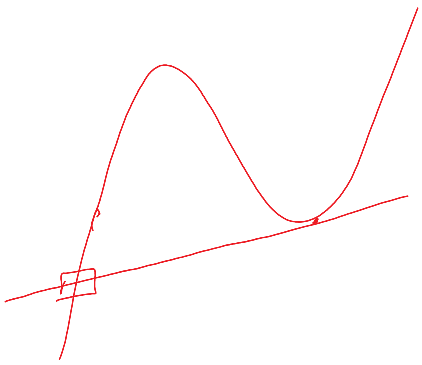
$$5 = 1 + \frac{1}{x^2}$$

$$4 = \frac{1}{x^2}$$

$$x^2 = \frac{1}{4}$$

$$x = \pm\sqrt{\frac{1}{4}}$$

$$x = \pm\frac{1}{2}$$



$$m_1 \times m_2 = -1$$

$$m_1 = \frac{-1}{m_2}$$

$$m_1 = \frac{1}{2} \qquad m_2 = -2$$

$$+\frac{1}{2} \Rightarrow -\frac{2}{1}$$

$$m_1 = \frac{2}{3} \qquad m_2 = -\frac{3}{2}$$