Completing the square and turning points

Sunday, 18 February 2018   7:24 pm

Work to be completed by the end of the lesson:

Completing the square	MM Ex. 3E	1cegh 2begh 3cdfgi 4bc 5ac(CAS) 6aef(CAS)

RECAP:

Remember, from a previous lesson when we have something in the following form we can find the turning point easily. We call the following **turning point form**.

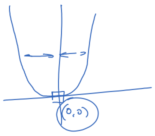
$$\boxed{y = a(x \pm b)^2 \pm k}$$

Dilation

Translation

Example:

$$y = 2(x - 3)^2 + 4$$



$\rightarrow 3$ $\uparrow 4$

$(3,4)$

When seeing this we can see that the turning point of the quadratic equation is (3,4).

Something to note/remember: A dilation does not affect the position of the turning point as it's simply stretching the graph.

DESMOS $y = x^2$ $y = 2x^2$ (TP)

How to complete the square:

E.g. Factorise the following function by completing the square: $y = \boxed{x^2 + 4x} - 3$

$(x+2)^2 = x^2 + 4x + 4$

$a(x \pm ?)^2 \pm ?$

$(x+2)^2 - 4 - 3 = (x+2)^2 - 7$

$TP = (-2, -7)$

$(x+2)^2 = (x+2)(x+2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

$(x+3)^2 = x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

E.g. Factorise the following function by completing the square: $y = \boxed{x^2 - 6x} + 10$

$(x-3)^2 = x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$

$(x-3)^2 - 9 - 10 = (x-3)^2 - 19$ $T.P = (3, -19)$

E.g. Factorise the following function by completing the square: $y = 3x^2 + 9x - 4$ $\div 3$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$y = 3\left(1x^2 + 3x - \frac{4}{3}\right)$

$= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{4}{3}\right]$

$\left(x + \frac{3}{2}\right)^2 = x^2 + \frac{3}{2}x + \frac{3}{2}x + \frac{9}{4}$

$= x^2 + 3x + \frac{9}{4}$

$= 3\left[\left(x + \frac{3}{2}\right)^2 - \frac{43}{12}\right]$

$-\frac{43}{12} \times \frac{3}{1} = -\frac{43}{4}$

$-\frac{9}{4} - \frac{4}{3} = \frac{-27 - 16}{12} = -\frac{43}{12}$

$= 3\left(x + \frac{3}{2}\right)^2 - \frac{43}{4}$

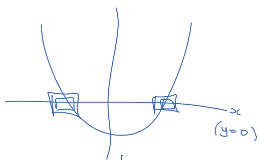
Solving equations:

E.g. $y = (x-3)^2 + 4$

$(x-3)^2 + 4 = 0$ -4

$(x-3)^2 = -4$

~~$(x-3) = \pm\sqrt{-4}$~~ $*$



E.g. $y = (x+4)^2 - 3$

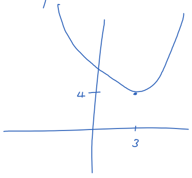
$(x+4)^2 - 3 = 0$ $(+3)$

$(x+4)^2 = 3$ $(\sqrt{\ })$

$x + 4 = \pm\sqrt{3}$

$x = -4 \pm \sqrt{3}$ $*$

~~Decimal~~



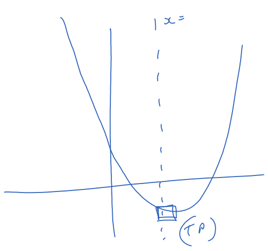
E.g. $y = 2(x-3)^2 + 6$

X

Axis of Symmetry

We already know that a quadratic has a line of symmetry down the centre.
The x-value happens to coincide with the mid-point of the two solutions to the quadratic equation.
When we find the x-value, we can find the y-value and hence the maximum or minimum of the quadratic

$$x = -\frac{b}{2a}$$



$y = (x-3)^2 - 6$

$T.P = (3, -6)$

Axis of symmetry $\Rightarrow$ $x = 3$

How to do this on the CAS