# Minimum Spanning Trees

Year 11
General Mathematics

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## Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Know what a tree is
- Know what a minimum spanning tree is
- Know how to use Prim's Algorithm to find the minimum spanning tree

## Recap

We have covered a lot of ground in these lessons all relating to Networks and graphs. We have covered a lot of language which, when put into you summary book, should make the questions relatively easy to answer.

We are now getting firmly into the content which is needed to do well at a Year 12 level. The work in this lesson forms a considerable part of the work in Year 12.

We look at what trees are and how we can use them to connect vertices in the shortest possible way.

## What is a tree?

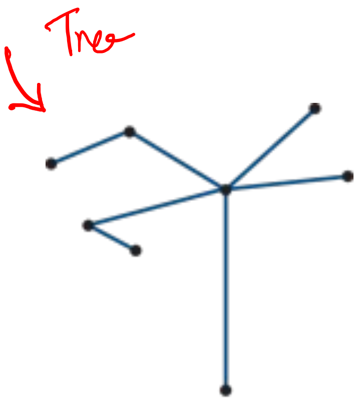
I couldn't help myself!!

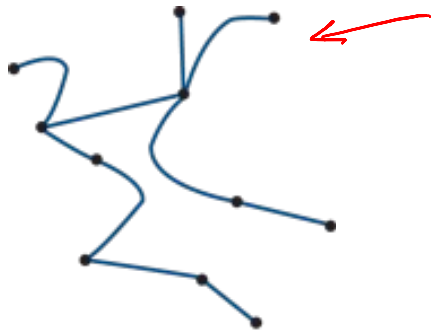
This is a tree, but it's not very useful for the topic we're going to do.

A tree, where graphs are concerned is, a connected graph that contains no cycles, multiple edges or loops.

It's important to know that a **tree can be part of a larger graph.**

Here are two examples of trees



# What is not a tree?

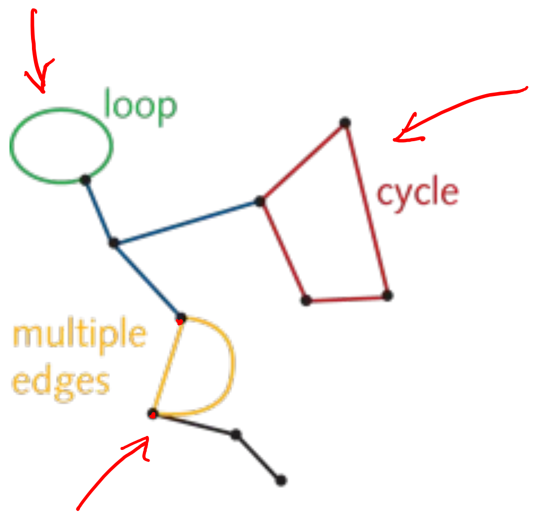
Remember, a tree cannot have loops, cycles or multiple edges!

So, the following is NOT a tree!

It's a complete mess …

It's also not a bird, but it does look like a person on a pogo stick, holding a laptop bag!

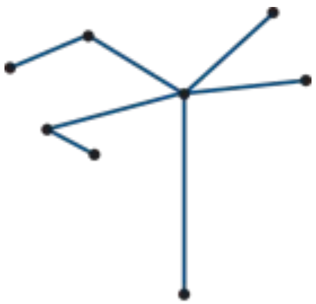
Now … try and unsee that!



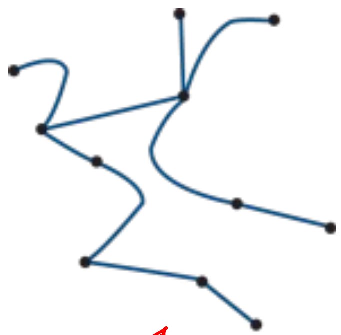
## Rule connecting edges and vertices

There is a rule which can be used for trees.

It's important to know that a tree with *n* vertices will have *n-1* edges



edges = 7
vertices = 8



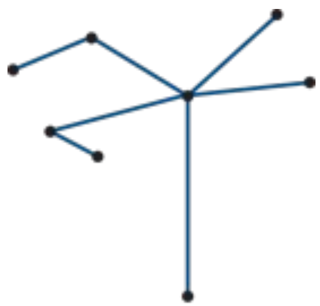
edges = 10
vertices = 11

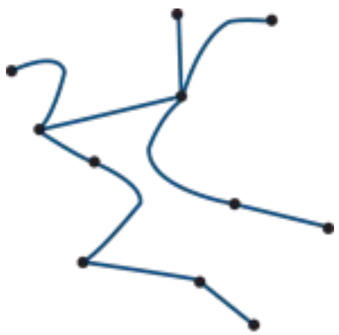
## Spanning Trees

A spanning tree is a tree which connects all the vertices in a connected graph.

The below could also be examples of spanning trees!

The edges are connecting all vertices.





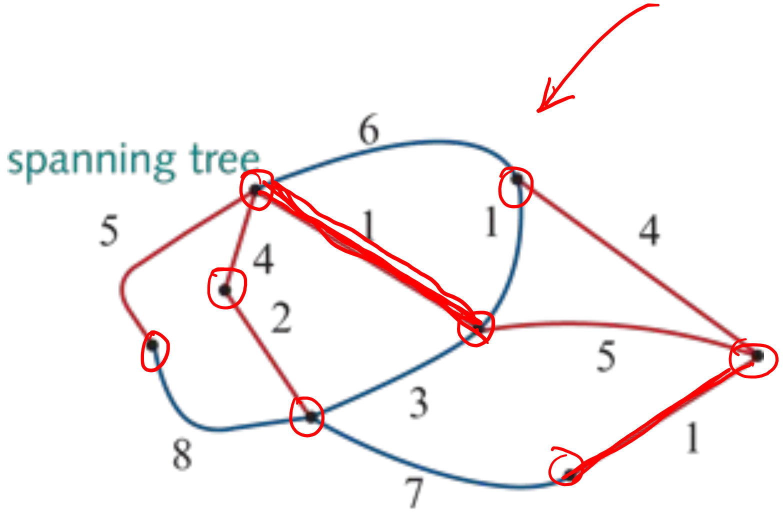
## Minimum Spanning Trees

OK. Perhaps we could have been more imaginative with the naming, but a **minimum spanning tree** is one where we connect all the vertices using the shortest possible edges.

There might be more than one edge connecting a diagram and, that allows us to choose the edges which are the shorted.

To be able to do this the **edges must all have weights**. Those weights will stand for something.

And example of a minimum spanning tree is shown below.



## Finding the minimum spanning trees

I have no idea who Prim is. But, they have come up with a way of finding the minimum spanning tree.

You basically follow an algorithm:

- Choose a starting vertex (any will do). Inspect the edges starting from this vertex and choose the one with the lowest weight. (If two edges have the same weight, the choice can be arbitrary.) You now have two vertices and one edge.
- Next, inspect the edges starting from the vertices. Choose the edge with the lowest weight. (If two edges have the same weight, the choice can be arbitrary.) You now have three vertices and two edges.
- Repeat the process until all the vertices are connected, and then stop. The result is a minimum spanning tree.

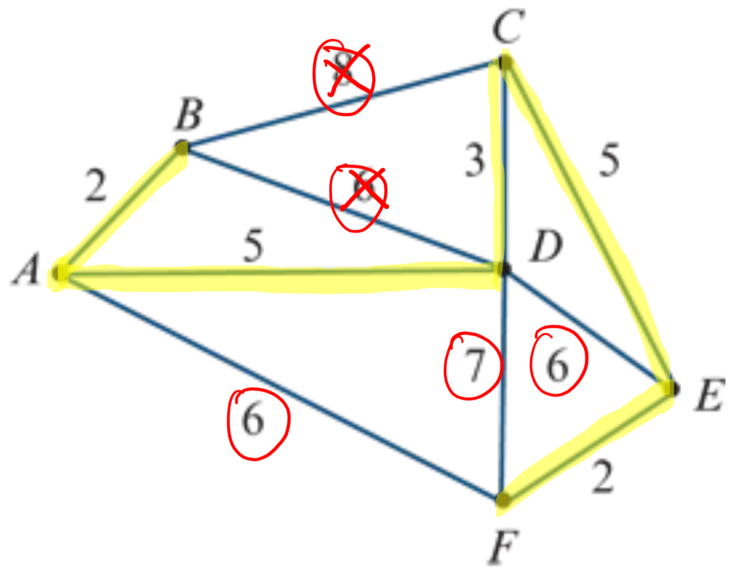
Let's have a look at some examples!

## Example: Applying Prim's algorithm

Apply Prim's algorithm to obtain a minimum spanning tree for the network shown, and calculate its length.

Remember:

- Choose a starting vertex (any will do). Inspect the edges starting from this vertex and choose the one with the lowest weight. (If two edges have the same weight, the choice can be arbitrary.) You now have two vertices and one edge.
- Next, inspect the edges starting from the vertices. Choose the edge with the lowest weight. (If two edges have the same weight, the choice can be arbitrary.) You now have three vertices and two edges.
- Repeat the process until all the vertices are connected, and then stop. The result is a minimum spanning tree.



## VCAA Example: Applying Prim's algorithm

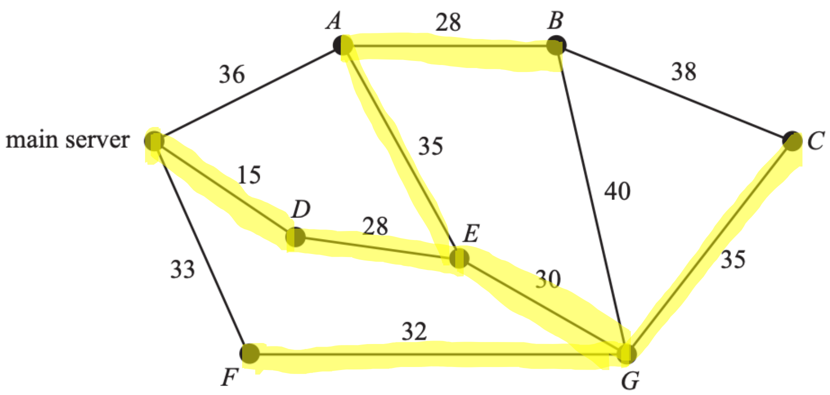
And here is a VCAA question which relates to Prims algorithm.

Questions are used, with permission.

VCAA 2019 Further Maths Exam 1, Question 5, Networks Module.

**Question 5**

The following diagram shows the distances, in metres, along a series of cables connecting a main server to seven points, *A* to *G*, in a computer network.




The minimum length of cable, in metres, required to ensure that each of the seven points is connected to the main server directly or via another point is

**A.** 175

**B.** 203

**C.** 208

**D.** 221

**E.** 236

15
28
35
28
30
32
35
20 3
3

## VCAA Example: Applying Prim's algorithm

And here is a VCAA question which relates to Prims algorithm. This one tests your understanding more than your ability to regurgitate.

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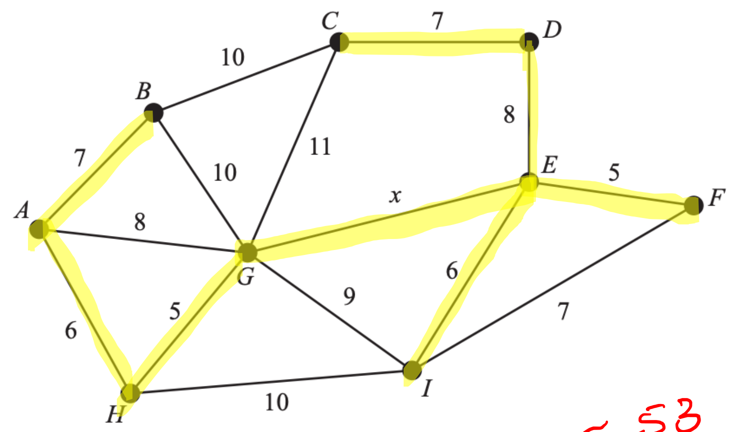
VCAA 2020 Further Maths Exam 1, Question 5, Networks Module.

**Question 5**

The network below shows the distances, in metres, between camp sites at a camping ground that has electricity.

The vertices *A* to *I* represent the camp sites.




The minimum length of cable required to connect all the camp sites is 53 m.

The value of $x$, in metres, is at least

**A.** 5
**B.** 6
**C.** 8
**D.** 9
**E.** 11

## Work to be completed

The following represents the minimum work which should be completed.

The more questions you answer from each exercise, chapter review and Checkpoints the better you chance of gaining an excellent study score in November.

**General Mathematics Units 1 and 2 Textbook**

Chapter 9
Exercise 9J Minimum Spanning Trees
Questions: All