Weighted graphs, networks and the shortest path problem



Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

- Know what a weighted graph is
- Know what a network is
- Know what it means to be the shortest path
- Know how to find the shortest path through a network

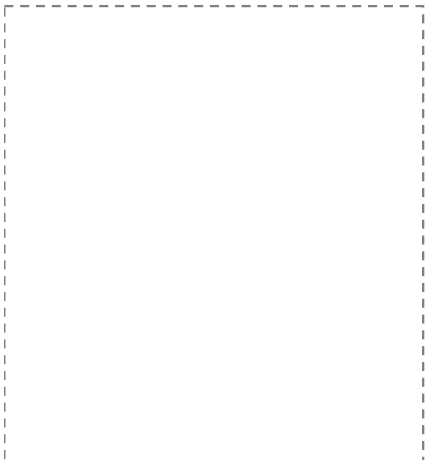
Recap

In previous lessons we have looked at lots and lots of language.

It's important that we can now talk confidently about:

- Walks
- Paths
- Trails
- Cycles
- Circuits
- Eulerian Trail
- Eulerian Circuit
- Hamiltonian Paths
- Hamiltonian Cycles

With all that out of the way, I think we've had enough of the language for now, let's actually use this in a practical way.



I used to work for the Post Office (UK)



True story. I hated it.

- I loaded mail onto a conveyer belt to be sorted by humans.
- I then got promoted to sort the mail into first and second class.
- I then got demoted to tying mail sacks with razor sharp metal clips.

Makes me grateful to be a teacher … But … I digress …

Imagine I am back, but a postie and want to deliver a lot of packages between:

- St Kilda
- Docklands
- Richmond
- Preston

There are lots and lots of routes I can take. If I really like to drive I could go via Geelong. But, that makes no sense! Hence, I want to find the shortest path between these places.

Weighted graphs



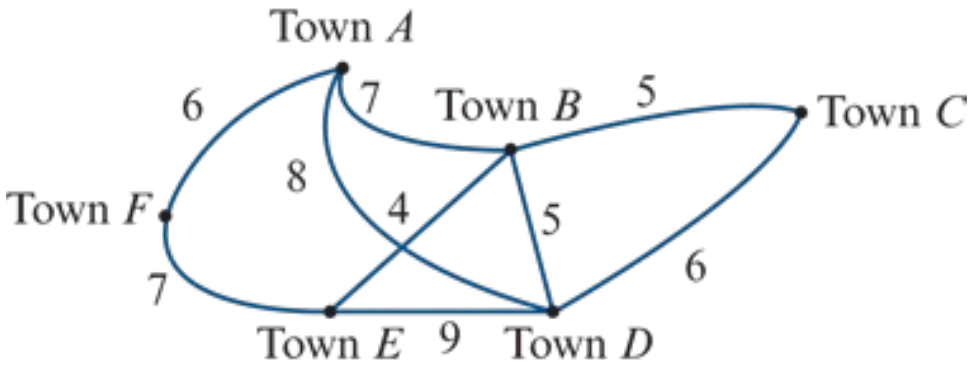
Weight. Very touchy subject. I'm wearing a corset to keep my youthful figure.
Mock me now … but trust me … it happens to the best of us.

Now, when I stand on a scale it gives me my weight. As a number.

When a graph has numbers on it we call it a **weighted graph**.

So, the following is an example of a weighted graph.

Because it's also modelling a real world situation, it's also called a **network**.



The numbers and the distances will be something I need to consider when travelling around my route delivering the mail.

Example: Interpreting a network

The network below is used to model the tracks in a forest connecting a suspension bridge (B), a waterfall (W), a very old tree (T) and a fern gully (F).

Walkers can enter or leave the forest through either Gate 1 $(G1)$ or Gate 2 $(G2)$.

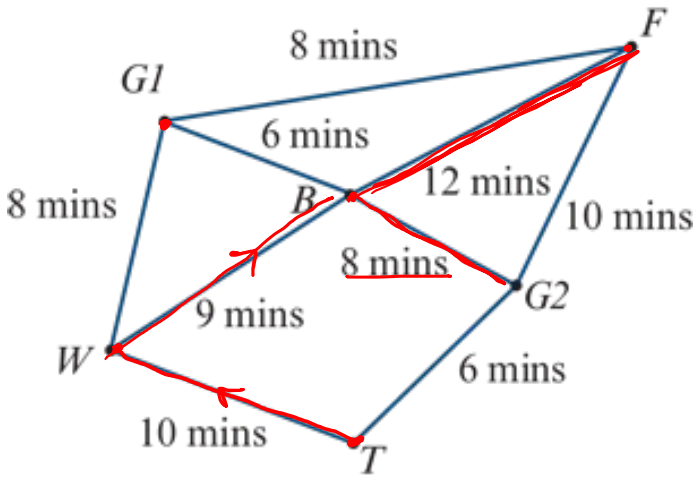
The numbers on the edges represent the times (in minutes) taken to walk directly between these places.

- How long does it take to walk from the bridge directly to the fern gully?
- How long does it take to walk from the old tree to the fern gully via the waterfall and the bridge?

B → G1 → F
6 8

B → G2 → F
8 10

10
9
12
31 mins



Finding the shortest path



We are always interested in finding the shortest path between things. This makes things more efficient and can reduce costs.

Think of laying water mains. We don't want to pay for lines to be laid which are not going to be used or are too long. This makes the cost of the project too much.

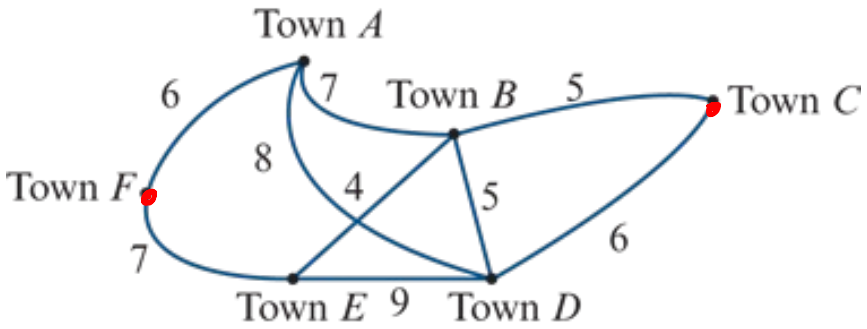
There are two ways to do it:

- Inspection
- Dykstra's Algorithm

We'll look at Inspection first.

Shortest path by inspection

Find the shortest route between Town C and Town F in the previous network opposite.



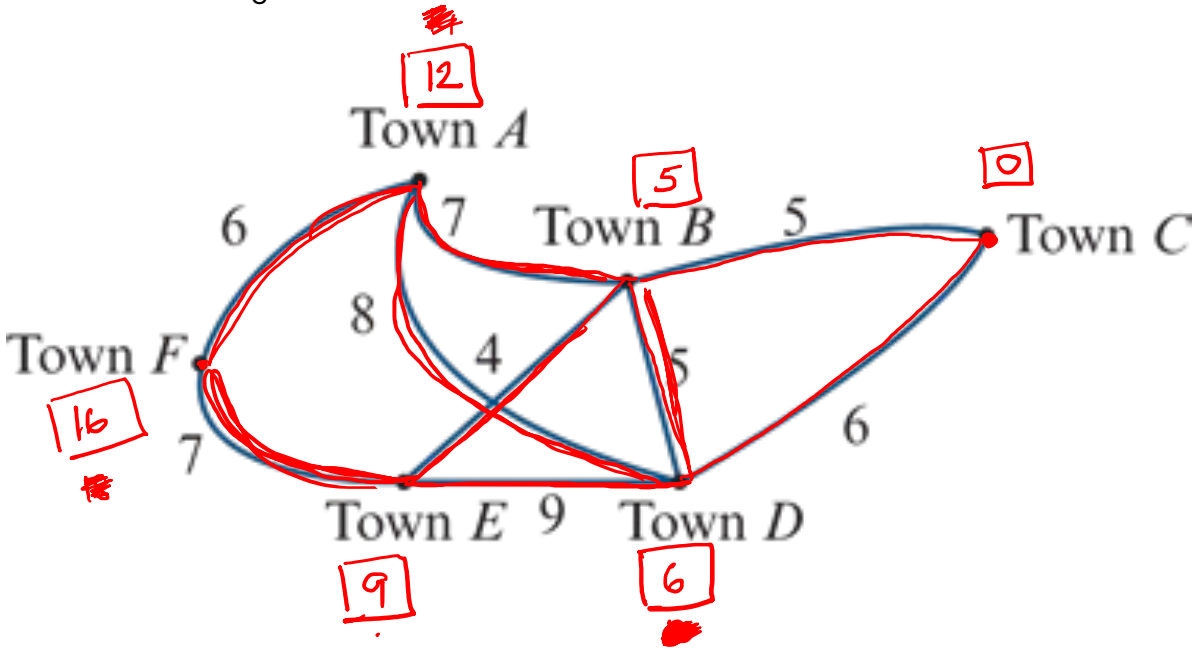
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Shortest path by using a Year 12 method

Find the shortest route between Town C and Town F in the previous network opposite.

C → F : 16 km

This is something which needs considerable practice.
You won't get it right first time, but it is worth doing.



Work to be completed

The following represents the minimum work which should be completed.

The more questions you answer from each exercise, chapter review and Checkpoints the better you chance of gaining an excellent study score in November.

General Mathematics Units 1 and 2 Textbook

Chapter 9
Exercise 9I Weighted graphs, networks and the shortest path problem
Questions: All