



Independent events

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means for events to be independent
- Understand how to show that two events are independent using probabilities



RECAP

We are nearing the end of this particular section of work relating to probability. Having spent some time in a previous lesson looking at conditional probabilities, we now spend time looking at Independent events.

What are independent events? And how can we check if two events are independent.



Back to tossing a coin again!

We know, from the previous section of work, that conditional probabilities can be calculated using the following formula:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

What happens if we look at the probability of getting a Head given that a Head has already happened when tossing two coins.

$$\begin{aligned}\Pr(H|H) &= \frac{\Pr(H \cap H)}{\Pr(H)} \\ &= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2}} \\ &= \frac{1}{2}\end{aligned}$$



Independent events

If we can show that when we have two events the occurrence of one event has no effect on the probability of another occurring then we say the two events are independent.

In other words:

$$\Pr(A|B) = \Pr(A)$$

We can take this formula and combine it with the one below to show something pretty interesting!

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$\cancel{\Pr(A|B)} = \frac{\Pr(A)}{\Pr(B)}$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

In other words, events A and B are only independent if $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$



Independent events

For events A and B with $\Pr(A) \neq 0$ and $\Pr(B) \neq 0$, the following three conditions are all equivalent conditions for the independence of A and B :

- $\Pr(A | B) = \Pr(A)$
- $\Pr(B | A) = \Pr(B)$
- $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

In the special case that $\Pr(A) = 0$ or $\Pr(B) = 0$, the condition $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ holds since both sides are zero, and so we say that A and B are independent.

Sometimes this definition of independence is referred to as **pairwise independence**.



Example 1

500 people were questioned and classified according to age and whether or not they regularly use social media.

The results are shown in the table.

Is the regular use of social media independent of the respondent's age?

Do you regularly use social media?

	Age < 25	Age ≥ 25	Total
Yes	200	100	300
No	40	160	200
Total	240	260	500

$$Pr(A \cap B) = Pr(A) \times Pr(B)$$

$$Pr(A < 25 \cap \text{Yes}) = \frac{200}{500} = \underline{\underline{0.4}}$$

$$\left. \begin{aligned} Pr(A < 25) &= \frac{240}{500} = 0.48 \\ Pr(\text{Yes}) &= \frac{300}{500} = 0.6 \end{aligned} \right\} \times = \underline{\underline{0.288}}$$

$$\therefore \text{Not ind as } Pr(A \cap B) \neq Pr(A) \times Pr(B)$$



Example 2

An experiment consists of drawing a number at random from $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5, 7\}$ and $C = \{4, 6, 8\}$.

a Are A and B independent?

b Are A and C independent?

c Are B and C independent?

$$a) \quad \Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

$$\frac{2}{8} = \frac{4}{8} \times \frac{4}{8}$$

$$\frac{1}{4} = \frac{1}{2} \times \frac{1}{2} \quad \checkmark$$

$\therefore A$ & B are indep.

$$b. \quad \Pr(A \cap C) = \Pr(C) \times \Pr(A)$$

$$\frac{1}{8} \neq \frac{3}{8} \times \frac{1}{2}$$

$\therefore A$ & C not ind.

$$c. \quad \Pr(B \cap C) = \Pr(B) \times \Pr(C)$$

$$0 \neq \frac{1}{2} \times \frac{3}{8}$$

$\therefore B$ & C not ind.



Example 3

Suppose we roll a die twice and define the following events:

A = the first roll shows a 4

B = the second roll shows a 4

C = the sum of the numbers showing is at least 10

a Are A and B independent events?

b What about A and C ?

	Die 1					
	1	2	3	4	5	6
Die 2	1					
	2					
	3					
	4					
	5					
	6					

$$a. \Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

$$\frac{1}{36} = \frac{1}{6} \times \frac{1}{6} \quad \checkmark$$

$\therefore A$ & B are independent

$$b. \Pr(A \cap C) = \Pr(A) \times \Pr(C) \quad \checkmark$$

$$\frac{1}{36} = \frac{1}{6} \times \frac{16}{36}$$

A & C ind



Example 4

Suppose that the probability that a family in a certain town owns a television set (T) is 0.75, and the probability that a family owns a station wagon (S) is 0.25. If these events are independent, find the following probabilities:

- a** A family chosen at random owns both a television set and a station wagon.
- b** A family chosen at random owns at least one of these items.

$$\begin{aligned} \text{a.) } \Pr(T \cap S) &= \Pr(T) \times \Pr(S) \\ &= 0.75 \times 0.25 = \underline{\underline{0.1875}} \end{aligned}$$

$$\begin{aligned} \text{b.) } \Pr(T \cup S) &= \Pr(T) + \Pr(S) - \Pr(T \cap S) \\ &= 0.75 + 0.25 - 0.1875 \\ &= \underline{\underline{0.8125}} \end{aligned}$$



Learning Objectives: Reviewed

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