



Conditional probability

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a conditional probability
- Know how to find conditional probabilities
- Know what the multiplication rule of probability is and when to use it.
- Know the law of total probability and when to use it
- Apply the learnings to a range of questions.



RECAP

In the last few lessons we have looked at the basics of how to use Venn Diagrams and Karnaugh Maps (Probability tables) to display and calculate various probabilities.

What we might not have been aware of is the relationship between events. We have met the idea that events can overlap. But what if the outcome of one event is conditional on the outcome of an event which came before.

This is the main focus of this part of the course.



Conditional probability

We are sometimes interested in calculating the probability of one event in light of whether another event has or has not already occurred.

A good way to think of this is to use the simple example of a coin toss.

What is the probability the a second toss of a coin will show a head if we know that the first toss was also a head?

We can express this using the following notation:

$$\Pr(\textit{second toss is a head} \mid \textit{first toss is a head})$$

Note the use of the vertical line.

It's read as "given that".

So the above would read "what is the probability that a second toss of a coin is a head **given that** the first toss was a head".

The order of these two conditions is really important!

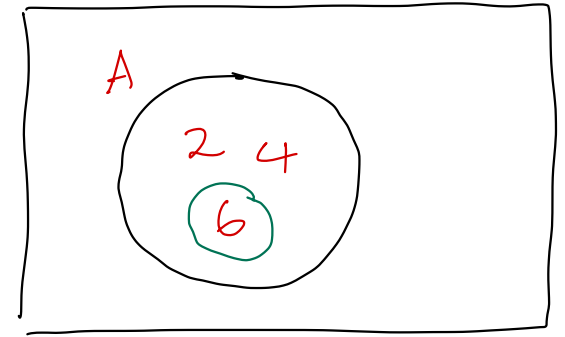


Example 1

Suppose we roll a fair die and define event A as 'rolling a six' and event B as 'rolling an even number'. What is the probability of rolling a six given the information that an even number was rolled?

$$\begin{aligned} \Pr(\text{rolling } 6 \mid \text{even rolled}) \\ &= \frac{1}{3} \\ &= \end{aligned}$$

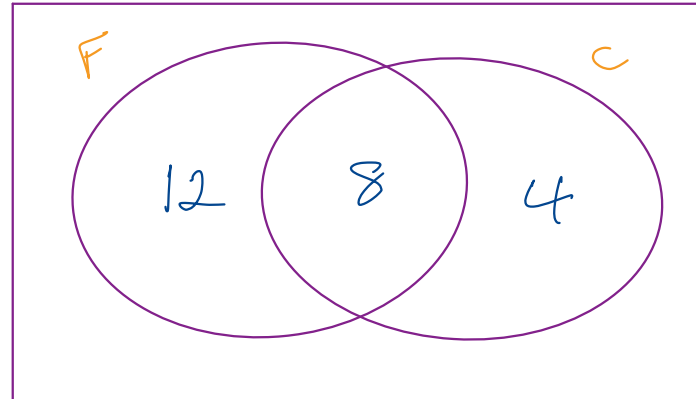
2, 4, 6



Example 2

In Stephen's class 12 students study Chinese, 20 study French, and 8 study both Chinese and French.

- a** Given that a student in his class studies Chinese (C), what is the probability that they also study French (F)?
- b** Given that a student in his class studies French, what is the probability that they also study Chinese?



$$\begin{aligned} \text{a) } Pr(F|C) &= \frac{8}{12} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

$$\text{b) } Pr(C|F) = \frac{8}{20}$$

$$\begin{aligned} &= \frac{4}{5} \\ &= \frac{2}{5} \end{aligned}$$



Example 3

500 people were questioned and classified according to age and whether or not they regularly use social media. The results are shown in the table.

Do you regularly use social media?

	Age < 25	Age $\geq$ 25	Total
Yes	200	100	300
No	40	160	200
Total	240	260	500

One person is selected at random from these 500. Given that the selected person is less than 25 years of age, what is the probability that they regularly use social media?

$$\begin{aligned} \Pr(\text{SM} | < 25) &= \frac{200}{240} \\ &= \frac{10}{12} \\ &= \frac{5}{6} \end{aligned}$$



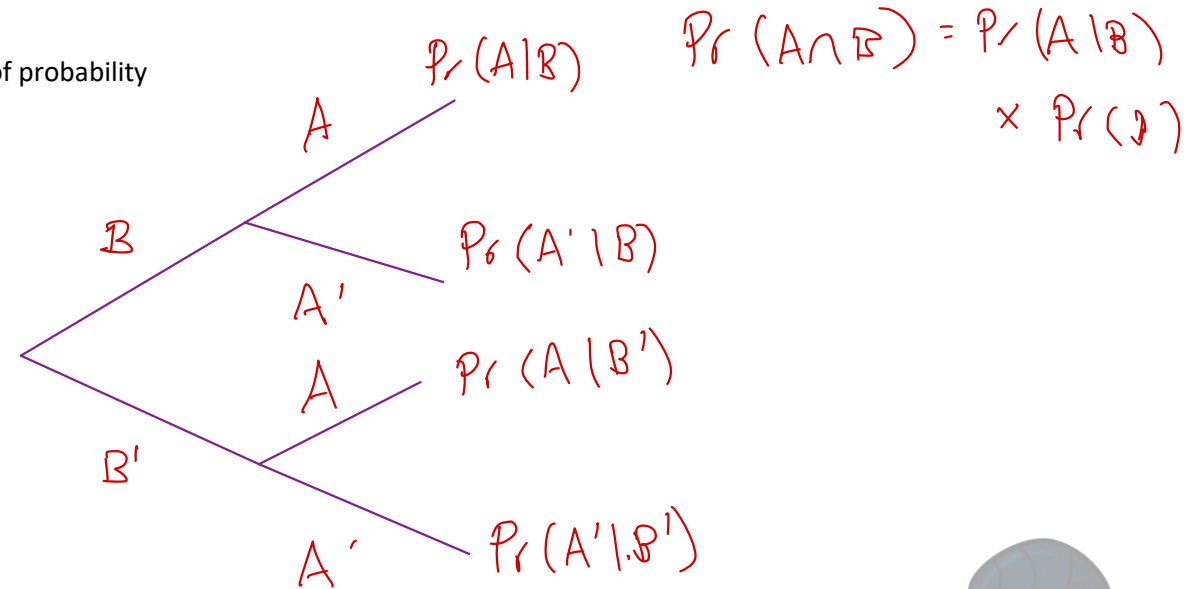
Multiplication rule of probability

A simple re-arrangement of the formula gives the below:

$$\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$$

This is called the multiplication rule of probability

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$



Example 4

In a particular school 55% of the students are male and 45% are female. Of the male students 13% say mathematics is their favourite subject, while of the female students 18% prefer mathematics. Find the probability that:

- a a student chosen at random prefers mathematics and is female
- b a student chosen at random prefers mathematics and is male.

$$P(M) = 0.55$$

$$P(F) = 0.45$$

$$P(P|M) = 0.13$$

$$P(P|F) = 0.18$$

$$\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$$

$$\begin{aligned} \text{a) } P(P \cap F) &= P(P|F) \times P(F) \\ &= 0.18 \times 0.45 \\ &= \underline{\underline{0.081}} \end{aligned}$$

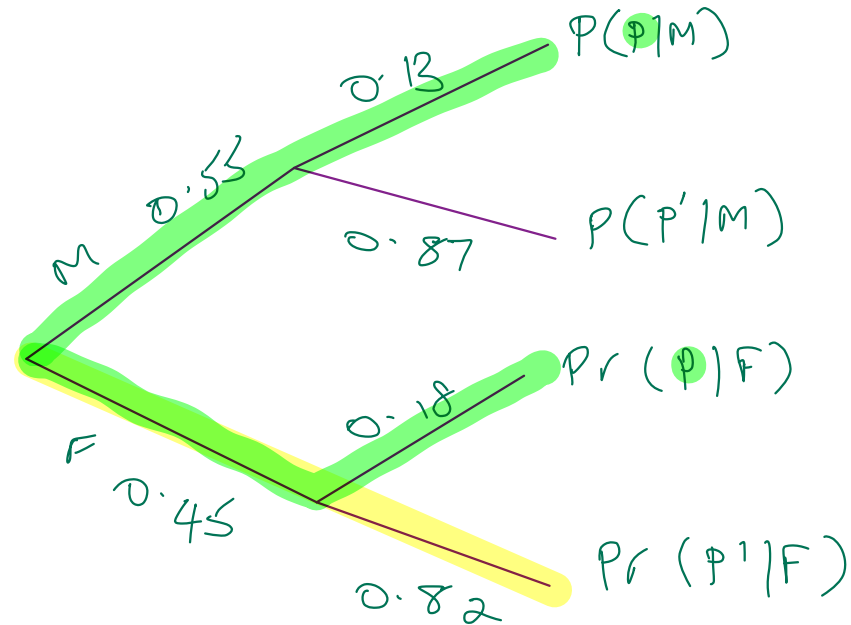
$$\begin{aligned} \text{b) } P(P \cap M) &= P(P|M) \times P(M) \\ &= 0.13 \times 0.55 \\ &= \underline{\underline{0.0715}} \end{aligned}$$



Example 5

Using the information from Example 21, construct a tree diagram and use it to determine:

- a the probability that a student selected is female and does not prefer mathematics
- b the overall percentage of students who prefer mathematics.



$$\begin{aligned} \text{a) } \Pr(P'|F) &= 0.45 \times 0.82 \\ &= 0.369 \end{aligned}$$

$$\text{b) } \Pr(P|M) = 0.55 \times 0.13$$

$$\Pr(P|F) = 0.45 \times 0.18$$

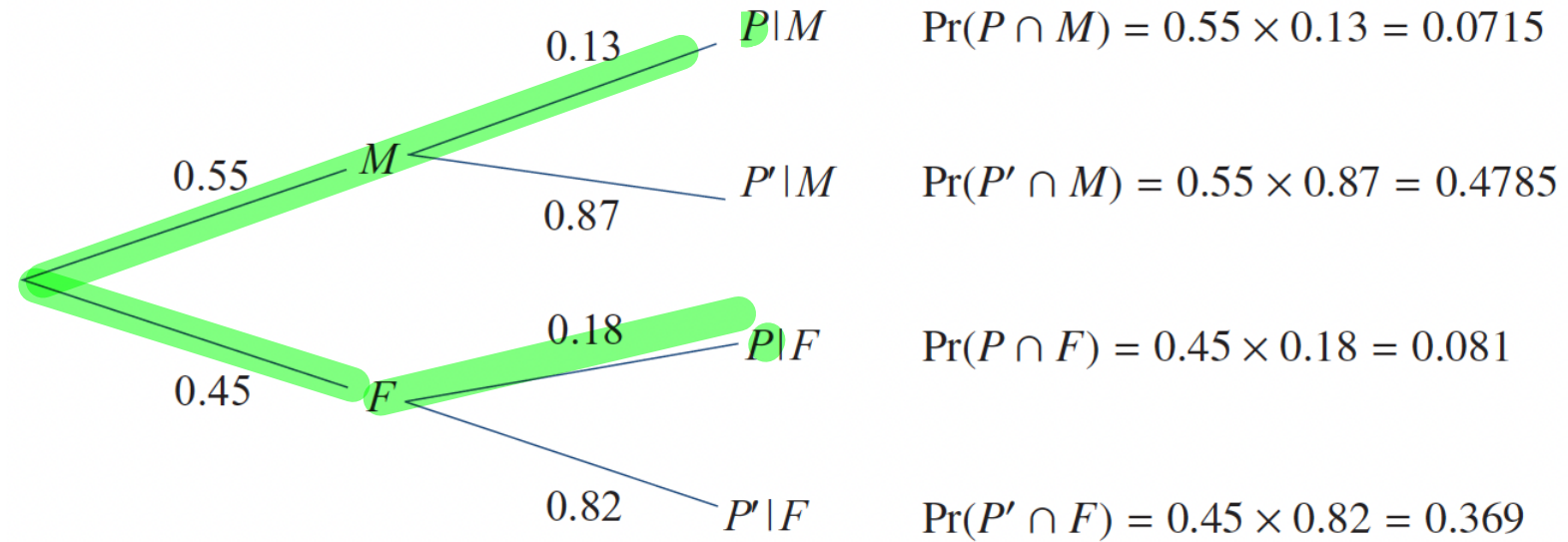
$$\therefore \Pr(P) = \underline{\underline{0.1525}}$$



The law of total probability

Using the ideas from the previous question, we can prove the following:

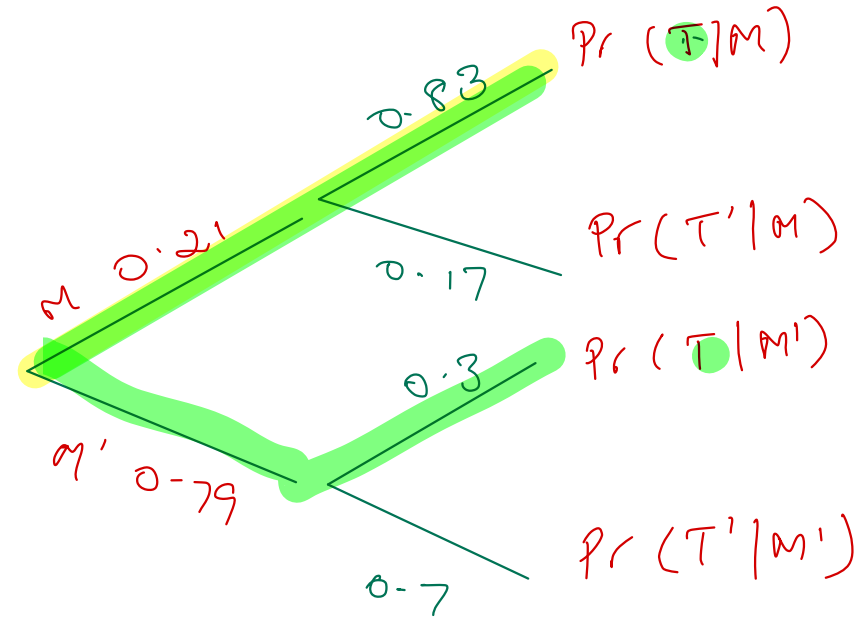
$$\Pr(A) = \Pr(B) \times \Pr(A|B) + \Pr(B') \times \Pr(A|B')$$



Example 6

In a certain town, the probability that it rains on any Monday is 0.21. If it rains on Monday, then the probability that it rains on Tuesday is 0.83. If it does not rain on Monday, then the probability of rain on Tuesday is 0.3. For a given week, find the probability that it rains:

- a** on both Monday and Tuesday **b** on Tuesday.



$$\begin{aligned} a, \quad Pr(M \cap T) &= 0.21 \times 0.83 \\ &= \underline{\underline{0.1743}} \end{aligned}$$

$$\begin{aligned} b, \quad P(T) &= 0.21 \times 0.83 \\ &\quad + 0.79 \times 0.3 \\ &= 0.1743 + 0.237 \\ &= \underline{\underline{0.4113}} \end{aligned}$$



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