Using arrays for two-step experiments



Year 9
Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- Understand what it means to be a two-step experiment
- Know the most appropriate way to display the results of a two-step experiment
- Know what it means to perform an experiment with and without replacement.
- Know how to calculate the probabilities associated with two-step experiments

Recap of past learning

In the previous lessons we have been looking at the basics of probability.

We have looked at how to find basic probabilities, how to use Venn Diagrams and two-way tables and how to use set notation. We move ever on with the more complicated parts of probabilities to look at things called two-step experiments.

What is a two-step experiment?

How would I show the results obtained when I roll a die twice? Or selecting two people from a group of people? It might be possible to do it in a sample space, but there are other ways to be able to do it which are more visual.

Humans are visual beings and we love pictures so we can use an **array** or table to show the results.

Throwing a coin and listing the outcomes.

Listing the outcomes is actually quite simple for two coins. It actually follows a binary pattern.

Outcomes:

HH
HT
TH
TT
↑ ↑

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↓ ↓ ↓
H HH
H HT
H TH
H TT
T H H
T H T
T T H
T T T

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Throwing a coin with a table (or array)

We can do this in a nicer way using a table (or array):

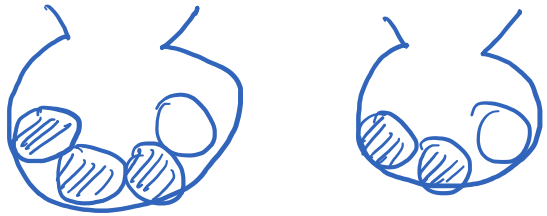
		First throw	
		H	**T**
Second throw	**H**	(H, H)	(T, H)
	T	(H, T)	(T, T)

$\mathcal{E} = \{(H,H), (T,H), (H,T), (T,T)\}$

With and without replacement

Many of the experiments we do are dependent on whether choices from a sample space are replaced or not.

We might choose to select a coloured ball from a bag. The probabilities of what will follow will be dependent on whether we replace the ball or not.



$Pr(\text{shaded}) = \frac{3}{4}$ ✓ $\frac{2}{3}$

CAT with replacement

Let's see what we can do with the letters CAT when we do an array of results both with and without replacement.

Imagine that we are selecting two letters from the word **CAT**.

		First letter		
		C	**A**	**T**
Second letter	**C**	(C, C)	(A, C)	(T, C)
	A	(C, A)	(A, A)	(T, A)
	T	(C, T)	(A, T)	(T, T)

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CAT with without replacement

Let's see what we can do with the letters CAT when we do an array of results both with and without replacement.

Imagine that we are selecting two letters from the word **CAT**.

		First letter		
		C	**A**	**T**
Second letter	**C**	x	(A, C)	(T, C)
	A	(C, A)	x	(T, A)
	T	(C, T)	(A, T)	x

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Examples

Two coins are tossed.
a) Draw a table to list the sample space
b) Find the probability of obtaining (H, T)
c) Find Pr(one head)

Note: The order is important!

2nd \ 1ST	H	T
H	(H,H)	(T,H)
T	(H,T)	(T,T)

$\Sigma = \Sigma$

$Pr(H,T) = \frac{1}{4}$

$Pr(\text{one head}) = \frac{2}{4} = \frac{1}{2}$



Example

Two letters are chosen at random from the word TREE without replacement
a) List the outcomes in a table
b) Find the probability that the two letters chosen are both E
c) Find the probability that at least one of the letters in an E.

Note: At least **includes** the number

1st.

2nd	T	R	E	E
T	x	(R,T)	(E,T)	(E,T)
R	(T,R)	x	(E,R)	(E,R)
E	(T,E)	(R,E)	x	(E,E)
E	(T,E)	(R,E)	(E,E)	x

12.

Pr(at least one E) $= \frac{10}{12} = \frac{5}{6}$

Pr(both E) $= \frac{2}{12} = \frac{1}{6}$