Planar graphs and Euler's formula

Year 11
General Mathematics

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

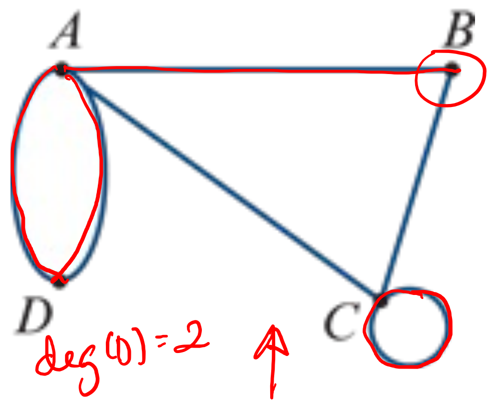
- Know what it means for a graph to be planar
- Know how to identify the faces (or regions) of a graph
- Know how to use Euler's formula to decide if a connected graph is planar or not.

Recap

We have been storming through the work to date and should now know the following:

- What an edge is
- What a vertex is
- What a loop is
- What multiple edges are
- What the degree of a vertex is
- How to represent a graph as an adjacency matrix
- What a connected graph is
- What a bridge is
- What an isomorphic graph is

We now move on to look at other types of graphs and something else with Euler found out.



Planar Graphs

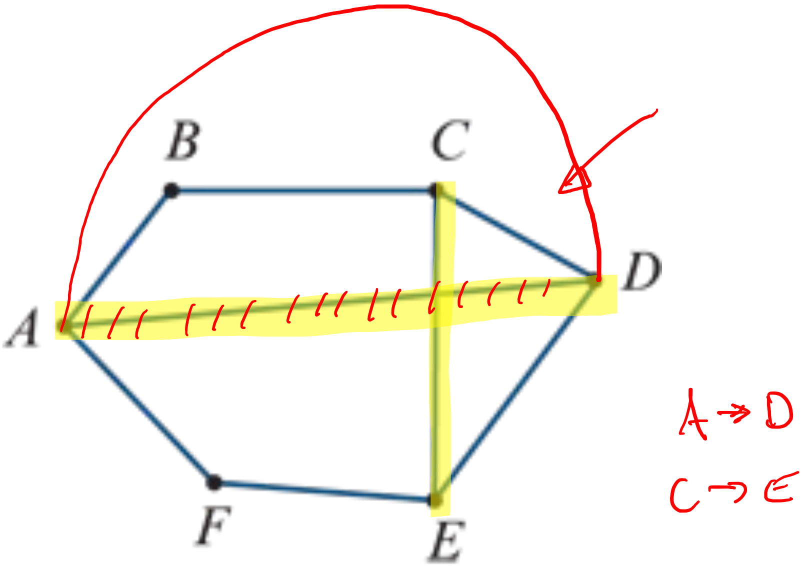
We have seen before that we can draw some graphs such they represent the same information but without any crossing edges.

It is pretty simple, in many cases, for is to use the idea of **elastic bands** to stretch one (or more) crossing edges so they no longer cross.

Look at the example shown.

The two highlighted edges are crossing.

Hence the graph is not planar.

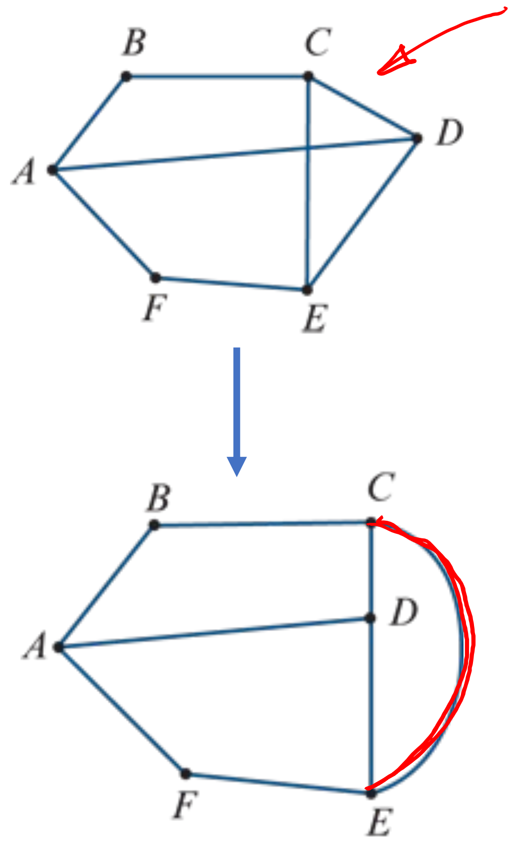


Planar Graphs

But, we can move the connection between C and E (or A and D).

Think of it as an elastic band which we can stretch!

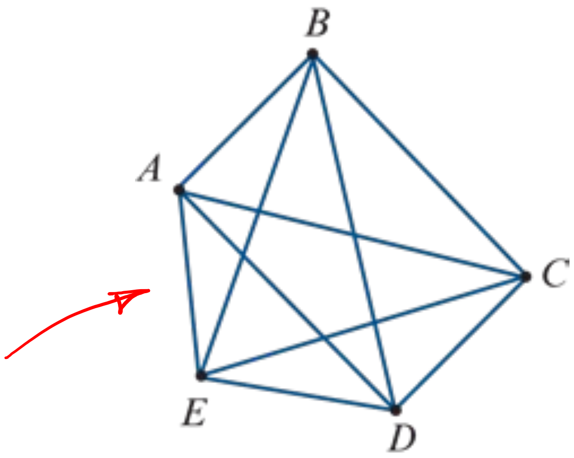
The second diagram shows the graph is now planar.



Not all graphs are planar

Can we apply the rubber band principle to the diagram shown?

Another one for the Studio Arts students ☺



More language: Faces (or regions)

Firstly, let's say that exams can called faces regions or regions faces.

Barry really is a pain in the butt!

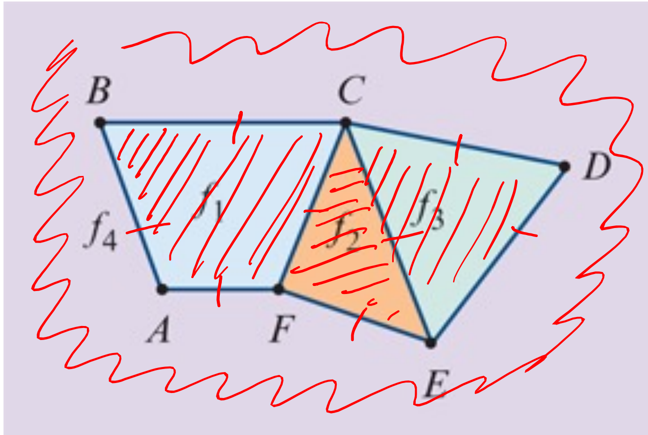
It's easier to show what a region is using a diagram.

The graph shown has:

6 vertices and 8 edges

We can see that if we were to colour in between the lines we could be able to do it three times inside the shape and once outside the shape.

This means the shape has 4 regions (or faces).



faces = 4

vertices = 6

edges = 8

$f + v - e$

$4 + 6 - 8 = \underline{\underline{2}}$

Euler's formula

Euler, who didn't seem to get out much as he was forever creating Maths formulas, came up with an interesting observation (and later formula) about planar shapes.

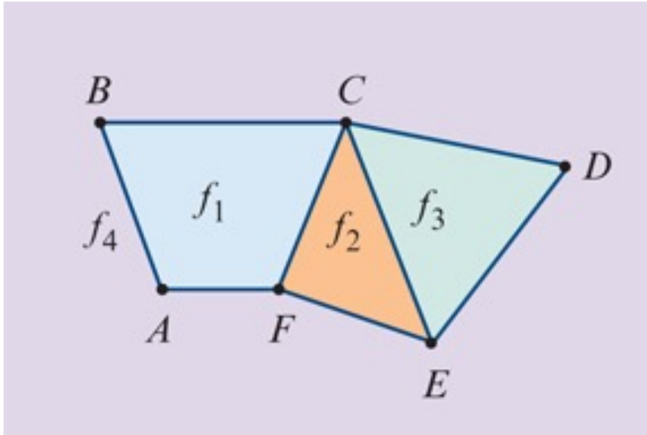
He found that:

Very fat minus elephants is always 2.

Hold on … that's my way of remembering! He said:

$$v + f - e = 2$$

Or if we add the vertices and faces and then subtract the number of edges, if it equals two then the shape is a **connected planar graph**.

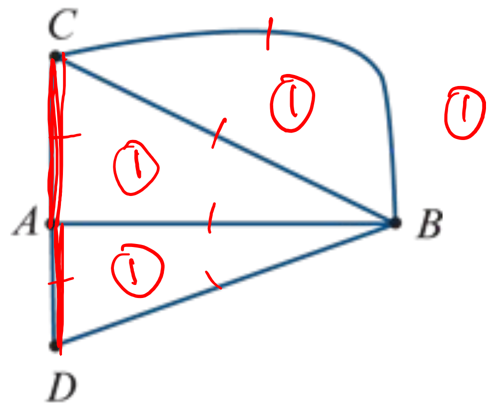


$$v + f - e = 2$$

Example 1

Consider the connected planar graph shown.

- Write down the number of vertices, v, the number of edges, e, and the number of faces, f.
- Verify euler's formula.



$v = 4$

$e = 6$

$f = 4$

$v + f - e = 2$

$4 + 4 - 6 = 2$ ✓

Example 2

A connected planar graph has four vertices and five edges. Find the number of faces.

$v = 4$

$e = 5$

$v + f - e = 2$

$4 + f - 5 = 2 \quad -4$

$f - 5 = -2 \quad +5$

$f = 3$

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Work to be completed

The following represents the minimum work which should be completed.

The more questions you answer from each exercise, chapter review and Checkpoints the better you chance of gaining an excellent study score in November.

General Mathematics Units 1 and 2 Textbook

Chapter 9
Exercise 9D: Planar graphs and Euler's formula
Questions: All