



Definite integrals by substitution

Year 12 Specialist Mathematics
Units 3 and 4

www.maffsguru.com

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Understand how to use the theory from the previous lesson and apply it to finding the solution to definite integrals.



RECAP

In the last lesson we looked at how we can use substitution to help us integrate more complex problems. All the examples we have dealt with so far have been indefinite integrals. This lesson will take the learnings and turn them into definite integrals.



Examples

Evaluate the following:

$$\int_0^4 3x\sqrt{x^2+9} dx$$

$$\int 3u^{1/2} \cdot \frac{du}{2} = \frac{3}{2} \int u^{1/2} du$$

$$= \frac{\cancel{3}}{\cancel{2}} \left[\frac{\cancel{2} u^{3/2}}{\cancel{2}} \right]_9^{25}$$

$$= \left[u^{3/2} \right]_9^{25}$$

$$= (125 - 27)$$

$$= \underline{\underline{98}}$$

$$\frac{\cancel{3}}{\cancel{2}} \left[\frac{\cancel{2}}{\cancel{2}} (x^2+9)^{3/2} \right]_0^4$$

$$u = x^2 + 9$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\int \cancel{3x} \cdot \sqrt{u} \cdot \frac{du}{\cancel{2x}}$$

$$u^{3/2} = \left(u^{1/2} \right)^3$$

$$= (\cancel{u^3})^{1/2}$$



Examples

Evaluate the following:

$$\int_0^{\pi/2} \cos^3 x \, dx = \int_0^{\pi/2} \cos^2 x \cdot \cos x \, dx$$
$$= \int_0^{\pi/2} (1 - \sin^2 x) \cdot \cos x \, dx$$

$$= \int (1 - u^2) \cdot du$$

$$= \left[u - \frac{u^3}{3} \right]_0^{\pi/2} = \left(1 - \frac{1}{3} \right) - \cancel{(0 - 0)}$$
$$= \frac{2}{3}$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$dx = \frac{du}{\cos x}$$



Examples

Evaluate the following:

$$\int_0^1 2x^2 e^{x^3} dx$$

$$\int 2 \cdot e^u \cdot \frac{du}{3} = \underline{\underline{\frac{2}{3}(e-1)}}$$

$$= \frac{2}{3} \int_0^1 e^u \cdot du$$

$$= \frac{2}{3} [e^u]_0^1$$

$$= \frac{2}{3} (e - 1)$$

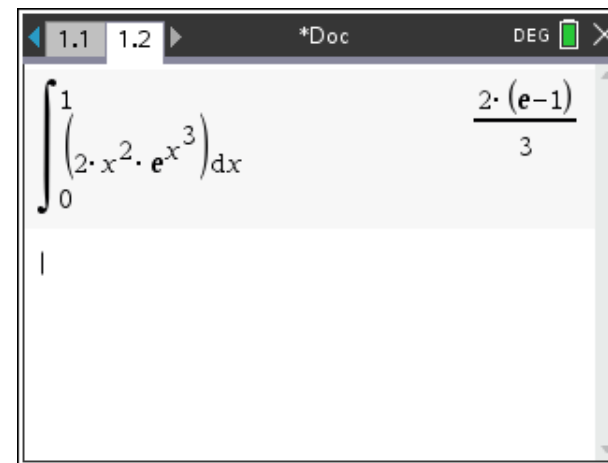
$$u = x^3$$
$$\frac{du}{dx} = 3x^2$$
$$dx = \frac{du}{3x^2}$$



Examples: Using the CAS

Evaluate the following:

$$\int_0^1 2x^2 e^{x^3} dx$$



A screenshot of a CAS interface. The top bar shows '1.1', '1.2', '*Doc', 'DEG', and a battery icon. The main display area shows the integral $\int_0^1 (2 \cdot x^2 \cdot e^{x^3}) dx$ on the left and the result $\frac{2 \cdot (e-1)}{3}$ on the right. Below the integral, there is a vertical cursor.



Learning Objectives: Revisited

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 3 and 4 Specialist Mathematics course.

- Understand how to use the theory from the previous lesson and apply it to finding the solution to definite integrals.



Making Maths Easy, Engaging Educational, Entertaining

Navigation: [Home](#)[Latest uploads](#)[Years 6 to 10](#)[VCE Courses](#)[Exam Solutions](#)[Buy Merchandise](#)

Why choose MaffsGuru?

I hate talking about myself.

So, here are some of the amazing comments I receive about the videos and content I produce followed by reasons to use the resource:

“I wish I watched your videos before naplan
— Overjoyed Cherry (youtube)”



VCAA exam questions

VCE lessons, where possible, include the use of past VCAA exam questions to



Professional Development

This resource isn't just meant for students. I hope it will be useful for teachers both new



Downloadable notes

Every lesson has downloadable notes. Whatever I write on the screen, you can download for



Respected Presenter

I currently present for Cambridge University Press and Nelson - as well as produce my own content for