Combining events

Year 11

Mathematical Methods

www.maffsguru.com

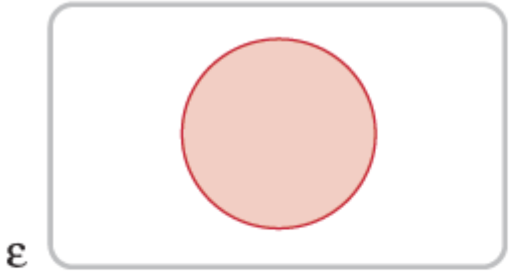
Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

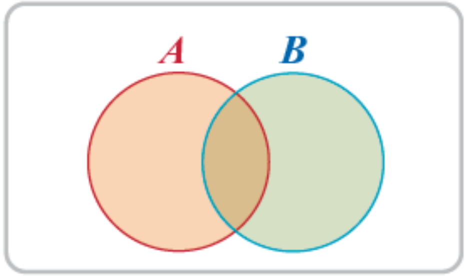
- Understand what it means to combine events in terms of probabilities
- Understand what the following means:
  - Empty set
  - Union
  - Intersection
  - Complement
  - Disjoint/Mutually exclusive
- Find the probability of combined events using
  - The addition rule

RECAP

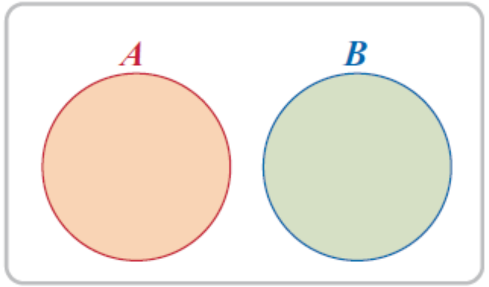
We need to remember that Mathematical Methods is filled with language. Generally, once you see through the language Methods becomes easier. So, here is a recap of the important language we are going to need to proceed with this section of the course. There are even diagrams!



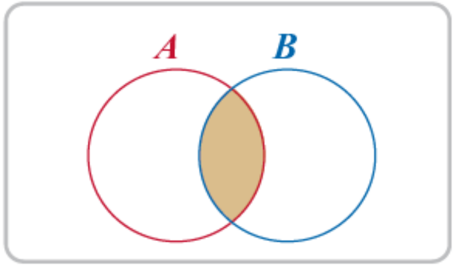
Empty Set ∅



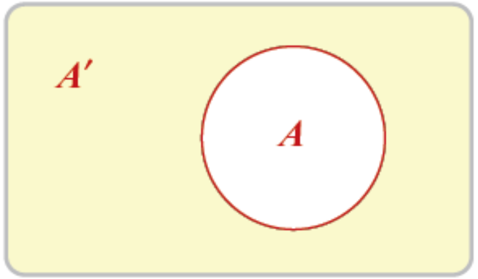
Union $A \cup B$ All elements in A or B (or both).



Disjoint/Mutually exclusive



Intersection $A \cap B$
Elements in both A and B.



Complement
Those elements NOT in the set.

Notation, notation and notation again

It seems to be really easy to trick people in methods.

A simple change in notation can lead students to misread the question and then give the wrong answer.

For example:

$$n(A)$$

This means the number of terms is set A. Not the probability.

$$Pr(A)$$

This means find the probability of an event happening.

Example 1

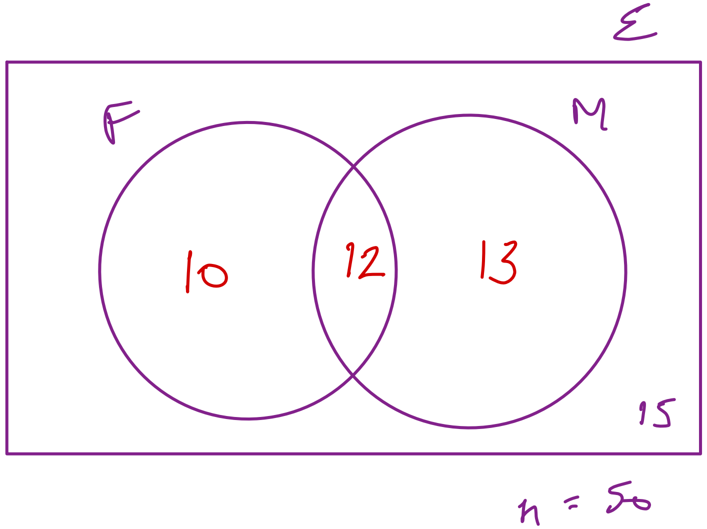
Fifty students were asked what they did on the weekends. A total of 35 said they went to football matches, the movies or both. Of the 22 who went to football matches, 12 said they also went to the movies. Show this information on a Venn diagram.

a How many students went to the movies but not to football matches?

b How many went neither to football matches nor to the movies?

a. 13

b. 15



a. $n(M \cap F') = \underline{\underline{13}}$

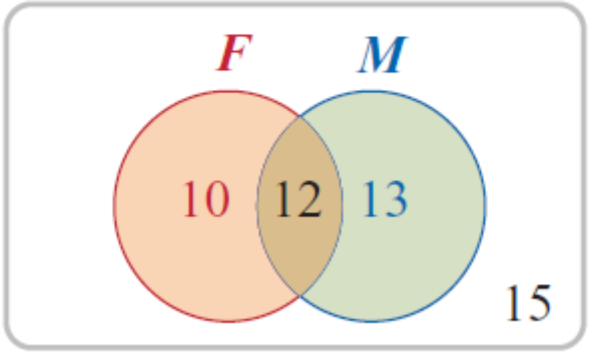
b. $n(M' \cap F') = \underline{\underline{15}}$

Example 2

Consider Example 12. What is the probability that a student chosen at random from this group of 50:

a went to the movies but not to football matches

b went neither to football matches nor to the movies?



$n = 50$

a. $\Pr(M \cap F') = \dfrac{13}{50}$

b. $\Pr(M' \cap F') = \dfrac{15}{50} = \dfrac{3}{10}$

The addition rule

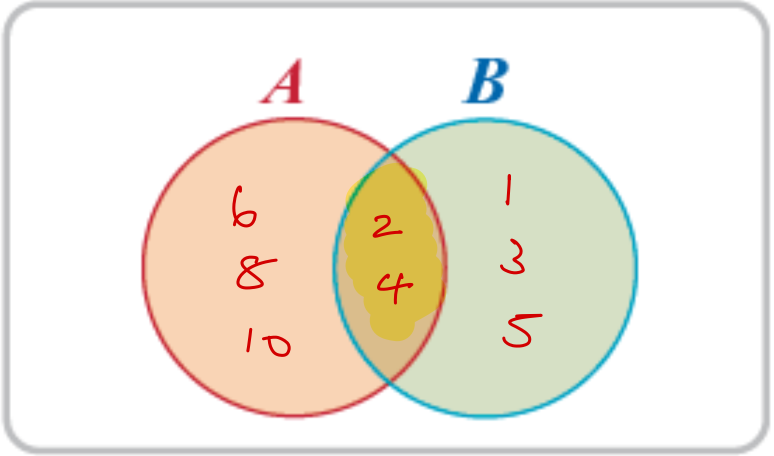
It is very important to take note of if there is an overlap between events. For example:

$$\Pr(A) = \{2, 4, 6, 8, 10\}$$
$$\Pr(B) = \{1, 2, 3, 4, 5\}$$

What is the probability of getting an even number in set A?
What is the probability of getting an even number in set B?

Normally in Maths, when we have an 'OR' condition we have told you to add them together. What we didn't tell you was that the data we used was **mutually exclusive**. There was no overlap.

If we were to add the two events together above, we would end up with a probability greater than one!



$$\Pr(\text{even } A \cap \text{even } B) = P(\text{even } A) + \Pr(\text{even } B)$$
$$= 1 + \frac{2}{5}$$
$$= 1.4$$

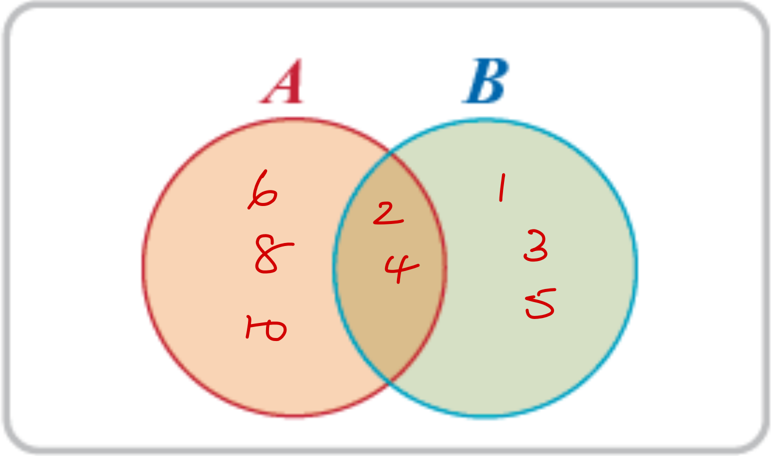
The addition rule

So, what we need to do is to work out the individual probabilities and then subtract the overlap as we don't want to **double count** the numbers.

This is more formally written as:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

This is the overlap.



$\Pr(\text{even } A \text{ or even } B) = 1 + \frac{2}{5} - \frac{2}{5}$

$= \underline{\underline{1}}$

Example 3

If one card is chosen at random from a well-shuffled deck, what is the probability that the card is a king or a spade?

$$\Pr(K \cap S) = \Pr(K) + \Pr(S) - \Pr(K \cup S)$$

$$\frac{1}{52} = \frac{4}{52} + \frac{13}{52} - \Pr(K \cup S)$$

$$\frac{1}{52} = \frac{17}{52} - \Pr(K \cup S)$$

$$\therefore \Pr(K \cup S) = \frac{16}{52} = \frac{8}{26} = \frac{4}{13}$$

Forwards and backwards

Now that we have a formula, we can basically give you, in a question, three of the four parts of the formula and ask you to work out the missing part!

For example:
Suppose $\Pr(A) = 0.3, \Pr(B) = 0.4\ and\ \Pr(A \cup B) = 0.5$. Find $\Pr(A \cap B)$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$0.5 = 0.3 + 0.4 - \Pr(A \cap B)$$

$$\Pr(A \cup B) = \underline{\underline{0.2}}$$

Draw a Venn Diagram

Suppose $\Pr(A) = 0.3$, $\Pr(B) = 0.4$ and $\Pr(A \cap B') = 0.2$. Find $\Pr(A \cup B')$.

Note: In many cases it makes sense to draw a Venn Diagram to help us find missing probabilities.

$\Pr(A \cup B')$

$$\Pr(A \cap B') = \Pr(A) + \Pr(B') - \Pr(A \cup B')$$

$$0.2 = 0.3 + 0.6 - \Pr(A \cup B')$$

$$0.2 = 0.9 - \Pr(A \cup B')$$

$$\therefore \Pr(A \cup B') = \underline{\underline{0.7}}$$

Understanding over regurgitation

Suppose $\Pr(A) = 0.28$, $\Pr(B) = 0.45$ and $A \subseteq B$. Find:

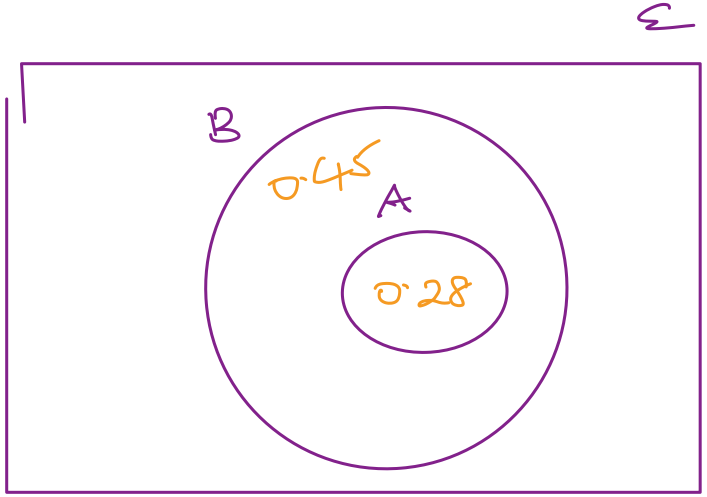
a $\Pr(A \cap B)$ **b** $\Pr(A \cup B)$

Note: Knowing a formula doesn't always help if you don't understand what the data looks like or is trying to tell you.

a. $\Pr(A \cap B) = 0.28$

b. $\Pr(A \cup B)$

$=$



$$\Pr(A \cap B) = \Pr(A) + \Pr(B) - \Pr(A \cup B)$$

$$\cancel{0.28} = \cancel{0.28} + 0.45 - \Pr(A \cup B)$$

$$\Pr(A \cup B) = 0.45$$

Learning Objectives: Reviewed

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to combine events in terms of probabilities
- Understand what the following means:
    - Empty set
    - Union
    - Intersection
    - Complement
    - Disjoint/Mutually exclusive
- Find the probability of combined events using
    - The addition rule

www.maffsguru.com