Multi-stage experiments

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what it means to be a multi-stage experiment
- Know what a tree diagram is and how to use it to display the outcomes from multi-stage experiments
- Know how to use a table to display the sample space of an experiment.

$\varepsilon = \{ \ , \ , \ . \ \}$

RECAP

We continue the review of Probability by building on the work which was covered before. Again, the work has been relatively simple so far.

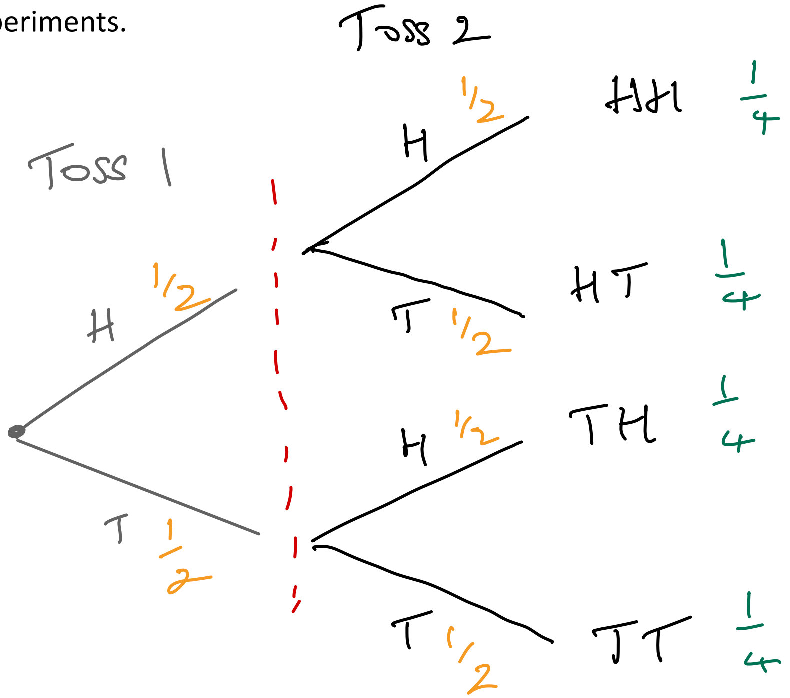
Multi-stage experiments

When we toss a coin once, it is clear there is going to be one of two outcomes; head or tail. The probabilities are also easy to find out. But what if we were to throw it twice? Or three times? What would the sample space look like? What would the probabilities look like? What would be the probability of getting a head, head and tail?

$\varepsilon = \{H, T\}$

Tree diagrams are really great at depicting multi-stage experiments.

Let's create one for two coin tosses.



$\Pr(HH) = 1/4$

$\varepsilon = \{HH, HT, TH, TT\}$

Example 1

$1 - \Pr(\text{no heads})$

$1 - \frac{1}{4} = \frac{3}{4}$

Sample Space

HH	$\frac{1}{4}$
HT	$\frac{1}{4}$
TH	$\frac{1}{4}$
TT	$\frac{1}{4}$

Find the probability that when a fair coin is tossed twice:

a one head is observed

b at least one head is observed

c both heads or both tails are observed.

a. $\Pr(\text{one head}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

b. $\Pr(\text{at least 1 head}) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$= \frac{3}{4}$

c. $\Pr(\text{HH or TT}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Using a table to display the sample space

Sample spaces can be displayed as a flat list or a table (for two stage events). Here is an example of a table when we throw two die. This would be preferable to drawing a tree diagram as it would be clearer.

Coin 2
die 6
12

There are 6 possible outcomes from Die 2

Die 1 \ Die 2	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Note the use of brackets and a comma. The order is also important.

There are 6 possible outcomes from Die 1

There are 36 possible outcomes in total

Question: Could we have worked out the total number of outcomes without drawing the table?

Example 2

Find the probability that when two fair dice are rolled:

a the same number shows on both dice (a double)

b the sum of the two numbers shown is greater than 10.

Die 2

Die 1	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a. $\Pr(\text{same num}) = \frac{6}{36}$

$= \frac{1}{6}$

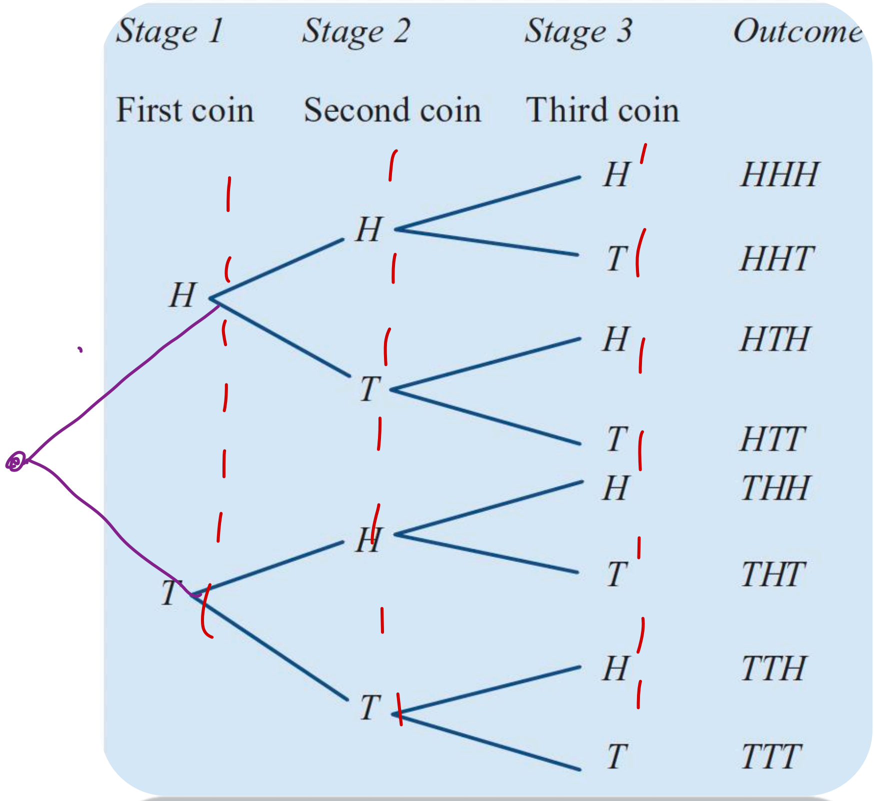
b. $\Pr(\text{sum} > 10) = \frac{3}{36} = \frac{1}{12}$

Tree diagrams are best for more than two stages

When there are more than two stages it makes sense to use a tree diagram (but these can be quite unwieldy too). Here is an example of tossing a coin three times.

When drawing a tree diagram, it's best to draw them in reverse.

Work out how many outcomes we are expecting and then reverse engineer it.



Example 3

Note: There are other ways of doing these without the use of a tree diagram.

Find the probability that when a coin is tossed three times:

a one head is observed

b at least one head is observed

c the second toss results in a head

d all heads or all tails are observed.

a. $Pr(1 \text{ head}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

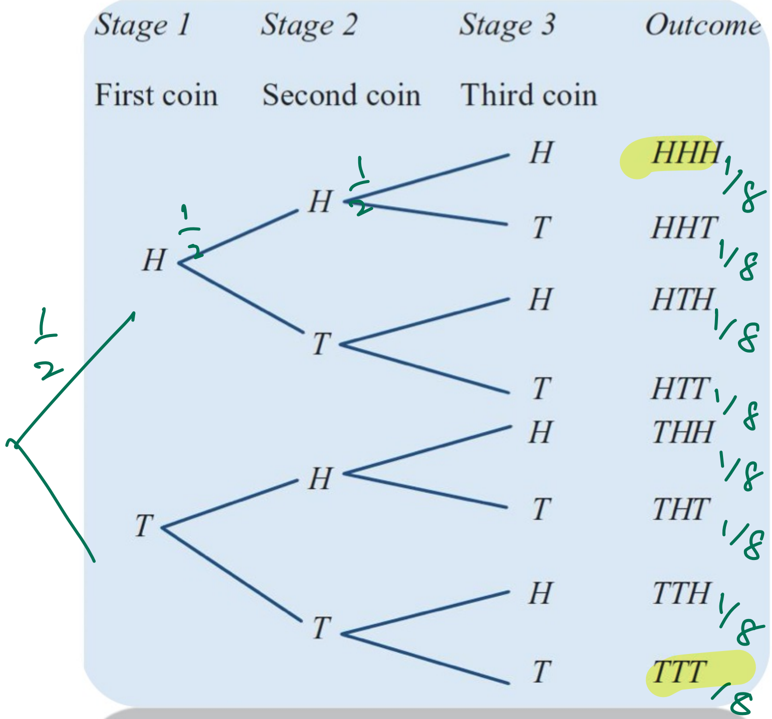
b. $Pr(\text{at least } 1 \text{ head})$

$= 1 - Pr(\text{no heads})$

$= 1 - 1/8 = 7/8$

c. $Pr(2\text{nd head}) = 4/8 = 1/2$

d. $Pr(HHH \text{ or } TTT) = 2/8 = 1/4$



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