Estimating probabilities

Year 11

Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Understand what a subjective probability is
- Understand how we might be able to collect probabilities from data
- Know what it means to be a relative frequency
- Know how to find probabilities
- Know what simulation is and when we might use it.
- Finding probabilities from areas

RECAP

In the last lesson we looked at the basics for probability. This was not particularly different from the work we have covered in Years 9 and 10 (here in Australia). Having the basics covered allows us to dig a little deeper into the Methods course.

Estimating probabilities: Subjective probabilities

In Australia (and the UK) we seem to be obsessed with the weather.

I have a widget on my computer screen designed to tell me the weather. I don't know why other than in case someone strikes up a conversation with me about the weather!

The one which I always wonder at is …

How do they know it's 50% chance of rain today? Where did that percentage come from?

This, to me, would be an example of a subjective probability. It is open to variation based on the person who is predicting it.

There is no "real" way to know if there is an actual chance of rain for 50% of the day.

Estimating probabilities: Probabilities from Data

Data is king! Does real data lie?

Well, that’s a story for another day … but when we have data, we can use it to find probabilities.

We can get this data from experiments. We can record the data and then use it to find the relative frequency to estimate the chances of it happening again in the future.

The following formula is helpful (and seems pretty familiar by now).

$$Relative\ frequency\ of\ event\ A = \frac{number\ of\ times\ event\ A\ occurs}{number\ of\ trials}$$

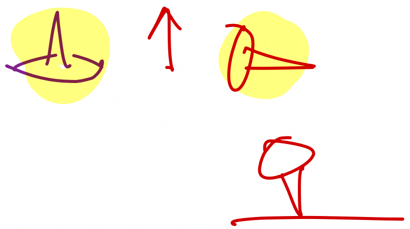
Is it a good idea to give a group of boys thumb tacks for an experiment?

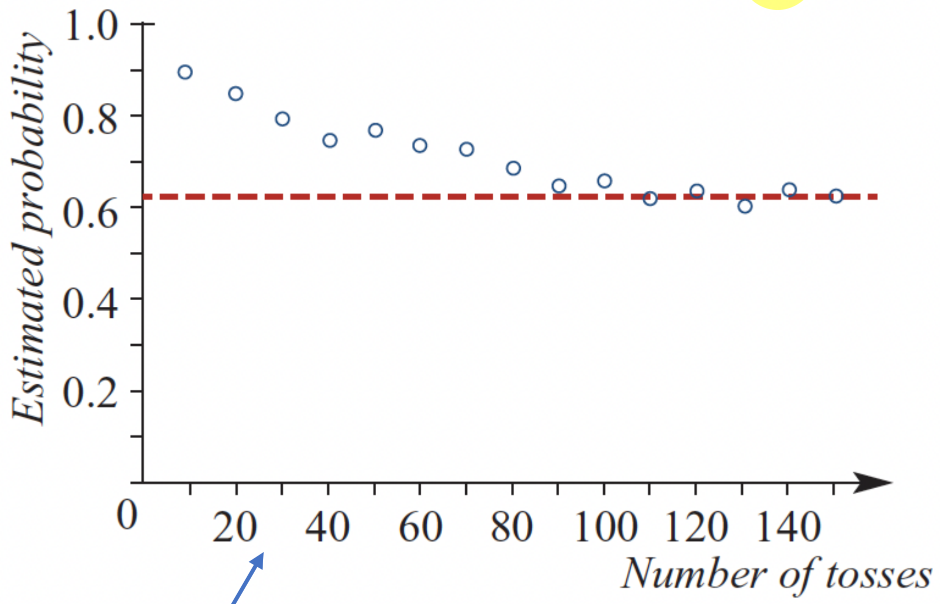
Short answer .. NO!

I used to perform a practical experiment where I would give a group of boys thumb tacks (drawing pins) and get them to drop it 100 times. We would then record how many times the thumb tack would land point up.

As we did more experiments, we would estimate the probability of getting a thumbs up and plot it on a graph.

That was the theory. But, boys being boys, they spend more time stabbing each other than dropping it.





Thankfully, Cambridge did the same thing and have the above graph

What we see is that with increasing numbers of throws, the estimated probability seems to start at 0.9 and then settle around 0.6.

We can use this to estimate the probability of a pin landing point up.

Large numbers of trials

When the number of trails is sufficiently large, the observed relative frequency of an event becomes close to the probability of the event occurring.

$$\Pr(A) \approx \frac{\textit{number of times event A occurs}}{\textit{number of trials}}$$

Example 1

In order to investigate the probability that a drawing pin lands point up, Katia decides to toss it 50 times and to count the number of favourable outcomes, which turns out to be 33. Mikki repeats the experiment, but she tosses the same drawing pin 100 times and counts 62 favourable outcomes.

a What is Katia's estimate of the probability of the drawing pin landing point up?

b What is Mikki's estimate?

c Which of these is the preferred estimate of the probability from these experiments?

d Based on the information available, what would be the preferred estimate of the probability?

a. $Pr(K) = \frac{33}{55} = \frac{66}{100} = 66\%$

$= 0.66$

b. $Pr(M) = \frac{62}{100} = 0.62 = 62\%$.

c. Mikki

d. $\frac{55 + 100 =}{33 + 62}$ $\frac{95}{155} = 0.62$ (2dp)

Simulation

I used to love this game! And did, for a short while, consider a career in city planning.

Sadly, I gave up that dream when I found that I couldn't quite work out how to get the water to flow fast enough and far enough for my cities. People died. A lot of people. Repeatedly.

Teaching is less stressful and dependent on water.

Sim City was a great example of simulation. But how does this relate to Mathematics?

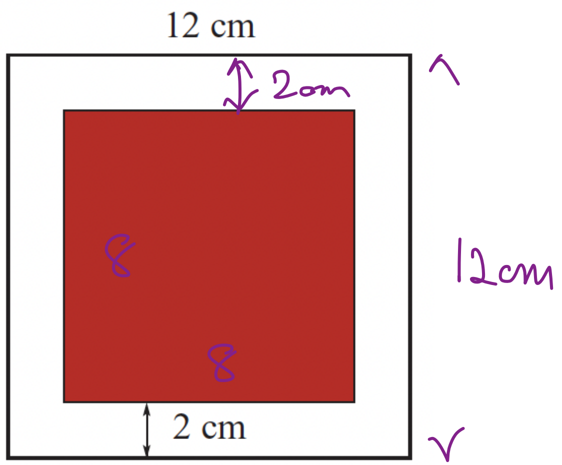
There are many instances where we can model real world examples on computers and collect data from it. This is potentially easier than actually going out into the real world … where there is water.

It might allow us to keep virtually repeating experiments and observing the outcomes.

Probabilities from area

Suppose that a square dartboard consists of a red square drawn inside a larger white square of side length 12 cm, as shown.

If a dart thrown at the board has equal chance of landing anywhere on the board, what is the probability it lands in the red area? (Ignore the possibility that it might land on the line or miss the board altogether!)



$$Pr(\text{red}) = \frac{\text{Area red}}{\text{Total area}} = \frac{64}{144}$$

$$\doteq 0.4\dot{4}$$

$$= 44.\dot{4}\%$$

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