Sample spaces and probability

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Be confident in knowing the following terminology:
  - Event
  - Random experiment
  - Sample space
  - Equally likely outcomes
  - Complementary events
- Be able to determine probabilities for equally likely outcomes
- Know how to find the probability of an event

RECAP

We have been looking at Probability since Year 7 and, to be honest, nothing really changes in Year 11 Mathematical Methods. We are still using the same terminology and finding probabilities is still pretty much the same. So, let's think of this section as a review of the work we have previously covered.

As is normal, the language of mathematics is really important as this is what tends to trip students up in exams.

Terminology

Sample space:
A list of all possible outcomes from an experiment. This can also be called a set of all possible outcomes. Contained within {}. Sample space has the symbol, $\mathcal{E}$

Greek letter, epsilon

Trial:
Single outcome e.g. Result from a single toss of a coin.

Random experiment:
One in which the results cannot be predicted.

Event:
Subset of a sample space. Can have more than one outcome e.g. Odd numbers from a standard die. Usually denoted with a capital letter.

Every year, in the VCE exam, people will tell me that an answer to a question relating to probability will be 1.67.

We blindly trust the calculator and forget to think about the reasonableness.

Sum of all probabilities
It is hopefully obvious that probabilities cannot sum to more than 1.

Example 1

A bag contains seven marbles numbered from 1 to 7 and a marble is withdrawn.

a Give the sample space for this experiment.

b List the outcomes (elements) of the event 'a marble with an odd number is withdrawn'.

a. $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7\}$

b. $A = \{1, 3, 5, 7\}$

Determining probabilities for equally likely outcomes

The most obvious item where the outcomes would be equally likely would be standard die.

Note: There are such things as biased die where one (or more outcome) can be more favourable.

$$\Pr(event) = \frac{\mathrm{n}(\text{event})}{n(sample\ space)}$$

We have seen this formula over and over again from Year 7.

Note: There are lots of examples where the probabilities will not be equally likely

Example 2

Suppose a number is drawn at random from the numbers 7, 8, 9, 10, 11, 12, 13, 14. What is the probability of choosing a prime number?

$$Pr(\text{Prime}) = \frac{3}{8}$$

Remember: A prime number is one with only two factors.

7
11
13

Example 3

A random experiment may result in 1, 2, 3 or 4. If $\Pr(1) = \frac{1}{13}$, $\Pr(2) = \frac{2}{13}$ and $\Pr(3) = \frac{3}{13}$, find the probability of obtaining a 4.

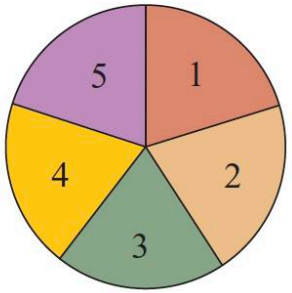
x	1	2	3	4
$\Pr(X = x)$	$\frac{1}{13}$	$\frac{2}{13}$	$\frac{3}{13}$	

$$\Pr(4) = \frac{7}{13}$$

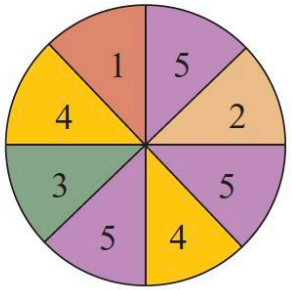
Example 4

Find the probability that each of the possible outcomes is observed for the following spinners:

a



b



1	1
2	1
3	1
4	2
5	3
	8

a. $Pr(1,2,3,4 \text{ or } 5) = \frac{1}{5}$

b. $Pr(1,2 \text{ or } 3) = \frac{1}{8}$

$Pr(5) = \frac{3}{8}$

$Pr(4) = \frac{2}{8} = \frac{1}{4}$

Compliment vs complement

One is where you write something really nice and flattering on my YouTube comments page (hint!) and the other is the negation of something.

In probability we can be asked for the complement of an event.

For example, the complement of **it's raining** would be **it's NOT raining**.

When adding the two probabilities together we will always get one (1).

Notation: $\Pr(A') = 1 - \Pr(A)$

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$\Pr(\bar{A}) = 1 - \Pr(A)$

Example 5

A card is drawn at random from a pack of 52 cards. What is the probability that the card is:

a not a heart

b not an ace?

a) $\Pr(H) = \frac{13}{52} = \frac{1}{4}$

$\Pr(H') = 1 - \Pr(H)$

$= 1 - \frac{1}{4} = \frac{3}{4}$

b) $\Pr(\text{not Ace}) = 1 - \frac{4}{52}$

$= \frac{48}{52} = \frac{24}{26} = \frac{12}{13}$

52

H | D | C | S

13 | 13 | 13 | 13

K

Q

J

10

9

8 →

7

6

5

4

3

2

A

Example 6

A random experiment may result in outcomes A, B, C, D or E, where A, B, C, D are equally likely and E is twice as likely as A. Find:

a $\Pr(E)$ **b** $\Pr(B')$

A	B	C	D	E
x	x	x	x	$2x$
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

$6x = 1$

$x = 1/6$

$\therefore \Pr(E) = \frac{2}{6} = \frac{1}{3}$

$\Pr(B') = 1 - \Pr(B) = 1 - 1/6$

$= 5/6$

Learning Objectives: Reviewed

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