Graph Theory Basics

Year 11
General Mathematics

www.maffsguru.com

Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Unit 1 and 2 General Mathematics course.

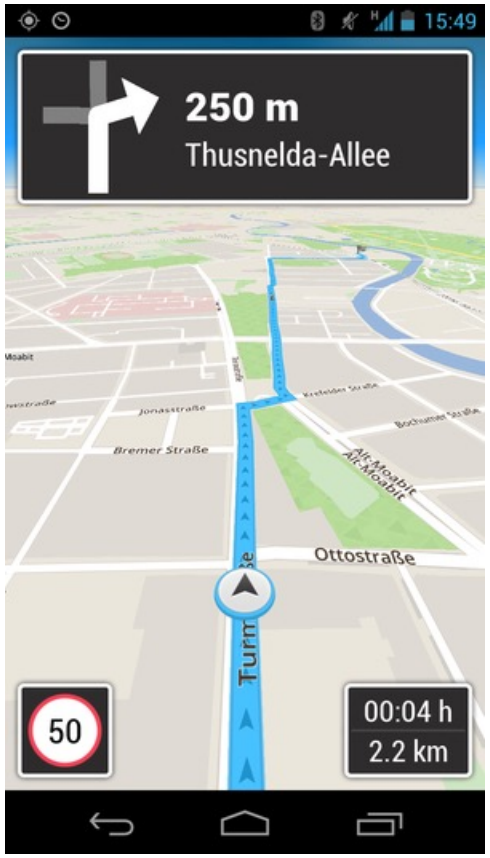
- Understand what a graph is
- Understand why a graph is important to us in the real world
- Understand what the following parts of a graph are:
    - Vertex
    - Edge
    - Degree
- Understand what it means for a graph to be planar

Recap

This is a completely new area of Mathematics for many of you and yet it is one of the most important. We use the concepts behind this in every day life. Find a car now without GPS enabled functions (be they visible or invisible).

How does a GPS know how to get from point A to B? Does it have every route possible programmed in?

This section of the work will be one of the most interesting but filled with lots more language.

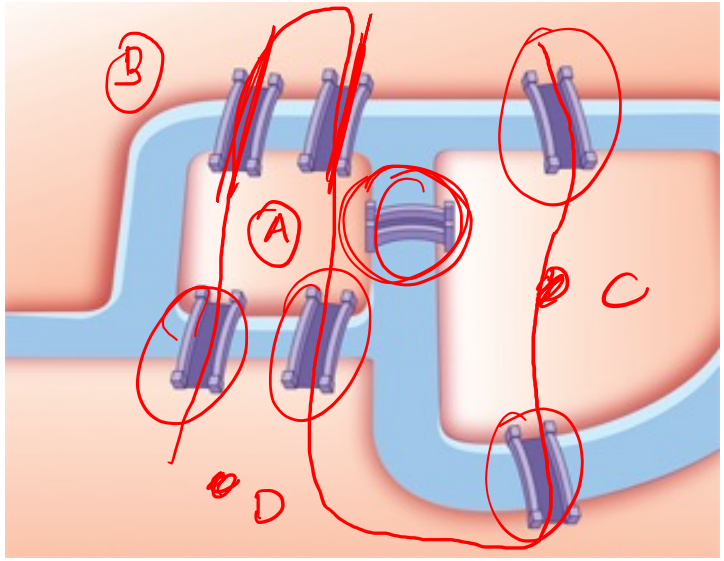


I want to go a travelling …

The centre of the old German city of Königsberg was located on an island in the middle of the Pregel River. The island was connected to the banks of the river and to another island by five bridges. Two other bridges connected the second island to the banks of the river, as shown.

Can a continuous walk be planned so that I can cross all the bridges only once?

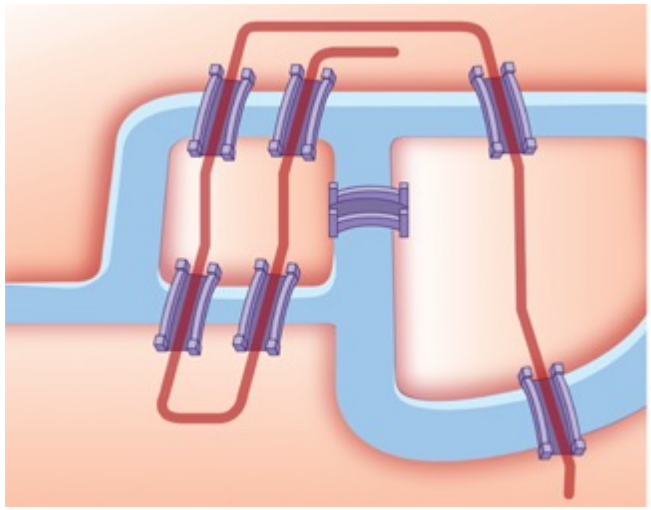
I'll wait ….

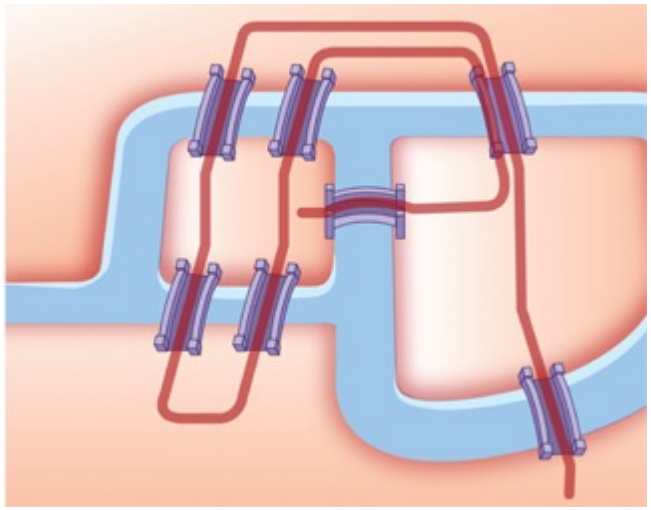


How many ways are there of trying this?

Here are just two of the ways of trying this problem.

Each time, I either have to cross one bridge more than once, or I cannot get over all the bridges.





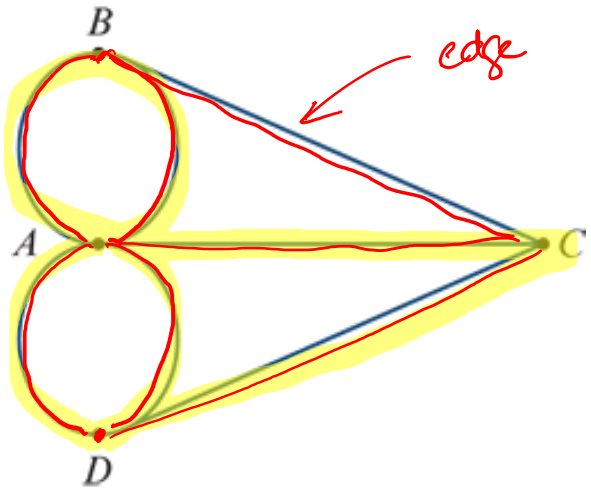
Making the problem simpler!

Well, the diagram at least.

If we look at using just lines and dots to stand for the places we want to go and the "roads" which we might travel we can get the following diagram. This is called a "graph".

Yup. I know. Looks nothing like the graphs we are used to drawing.

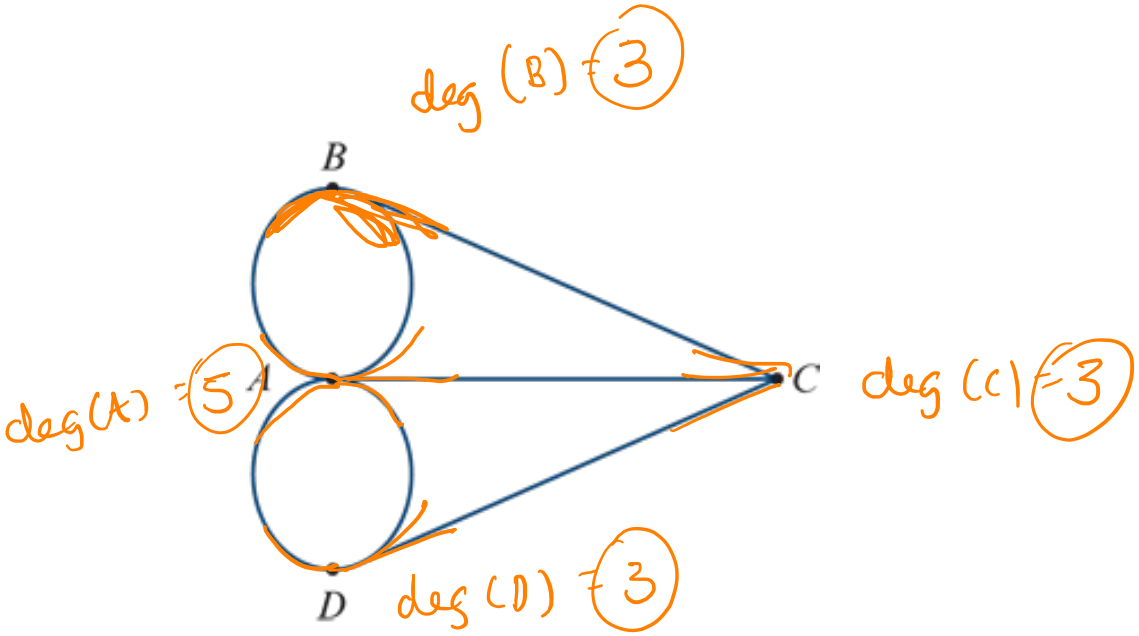
Good old Barry!



Language of graphs

There is now some important language we need to know:

Vertex: The dots on the graph
Edge: The roads connecting the dots.



Euler was a clever dude…

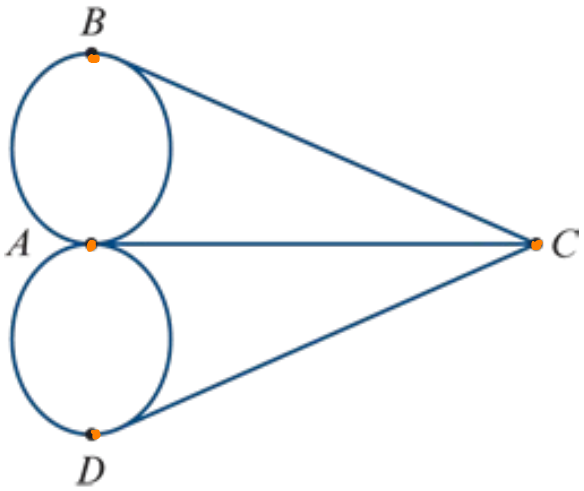
Leonhard Euler
15 April 1707 – 18 September 1783
A Swiss mathematician, physicist, astronomer, geographer, logician and engineer who made important and influential discoveries in many branches of mathematics, such as infinitesimal calculus and graph theory, while also making pioneering contributions to several branches such as topology and analytic number theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function.[4] He is also known for his work in mechanics, fluid dynamics, optics, astronomy and music theory.[5]

Extracted from Wikipedia

Euler noticed something ...

If we take the “degree” of a vertex which is the number of roads leading into or out of a vertex, we find something very interesting.

Remember:
Vertex: The dots on the graph
Edge: The roads connecting the dots.

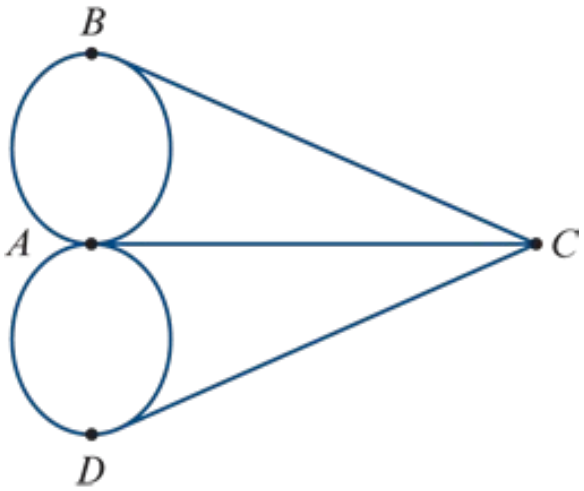


When the degree are all odd ...

We cannot travel along every road without having to go down a road we have already travelled if the degrees of the vertices are all odd.

WOW!

This, of course leads onto more stuff later on in Year 12 but, for now, we are happy that we can label a vertex, edge and find the degree.



Work to be completed

The following represents the minimum work which should be completed.

The more questions you answer from each exercise, chapter review and Checkpoints the better you chance of gaining an excellent study score in November.

General Mathematics Units 1 and 2 Textbook

Chapter 9
Exercise 9A: Graph Theory Basics
Questions: All (there are only two questions)