

## Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to apply the amortisation process.
- To be able to construct an amortisation table.
- To be able to analyse an amortisation table for a reducing balance loan.
- To be able to read and interpret an amortisation table for an annuity to find the interest rate.
- To be able to interpret and construct an amortisation table for a compound interest investment with additions to the principal.


## Recap

It seems to appear that the following formula is pretty much all we need to know for General Maths.

$$
V_{0}=\text { Principal, } \quad V_{n+1}=R \times V_{n} \pm D, \quad \text { where } R=1 \pm \frac{r}{100}
$$

However, we are now going to diverge for a small while and look at something which used to be delivered to everyone each month.

It was something we all spent ages looking over to make sure that we hadn't had money stolen from us.
We called it a bank statement but General Maths wants it to be called an Amortisation Table.

HSBC (z) UK


$$
\begin{aligned}
& \text { Contact tel } 034574044
\end{aligned}
$$

$$
\begin{aligned}
& \text { Your Statement }
\end{aligned}
$$


internatonal Bank Account Number

$\begin{array}{lll}\begin{array}{lll}\text { Sortcode } \\ \text { 40-25-2 }\end{array} & \begin{array}{l}\text { Account Number Shect Number } \\ 123456979\end{array} & 16\end{array}$

Paid out Paid in Balance
9.000
$\underset{50.50 .57}{590.5}$

## Amortisation tables

These are effectively ways to show the current state of a loan or investment. At the moment we will look at one for a loan.

The most important thing to note is:
Principal amount of the loan
Principal reduction $=$ Payment - Interest

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 1000.00 |
| 1 | 250.00 | 12.50 | 237.50 | 762.50 |
| 2 | 250.00 | 9.53 | 240.47 | 522.03 |
| 3 | 250.00 | 6.53 | 243.47 | 278.56 |

Monthly (or other) payment

How much interest has been charged on the previous balance

How much the loan has been reduced by

## Amortisation tables: Finding the rate of interest

A common question in exams is to find the rate of interest from an Amortisation table.
We can use the formula:

$$
\text { Interest per compounding period }=\frac{\text { Amount of interest charged }}{\text { Balance from previous payment }} * 100 \%
$$

$$
\begin{aligned}
& 1.25 \% \text { of } 278.56 \\
& 1.25 \div 100 \times 278.56
\end{aligned}
$$

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 1000.00 |
| 1 | 250.00 | 12.50 | 237.50 | 762.50 |
| 2 | 250.00 | 9.53 | 240.47 | 522.03 |
| 3 | 250.00 | 6.53 | 243.47 | 278.56 |

So, for the previous example:

$$
\text { Interest }=\frac{12.50}{1000} * 100 \%=1.25 \% \text { per compounding period }
$$



It is very important to know that this is the rate per compounding period.

## Amortisation tables: What you will be asked for

It is common that they ask you to complete lines of an amortisation table. This is more common in SACs than exams, but
it has been asked in exams before.
They would ask you to find:

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 1000.00 |
| 1 | 250.00 | 12.50 | 237.50 | 762.50 |
| 2 | 250.00 | 9.53 | 240.47 | 522.03 |
| 3 | 250.00 | 6.53 | 243.47 | 278.56 |

- Interest payments and rates of interest
- Final payments (which might be smaller than normal)
- Values of principal reductions given payments and interests
- Balances at any point given past balance and principal reductions


Example: Amortisation tables
Flora borrows $\$ 20000$ at an interest rate of $8 \%$ per annum, compounding annually. She

$$
r=8 \%
$$ makes annual payments of $\$ 2500$.

a State the principal of the loan. $\$ 20000$
b Calculate the initial interest charged on the principal.
c Determine the impact of the first annual payment to find the principal reduction.
d Calculate the new balance.
e Complete the row in the table below with your calculations.

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | :--- | :---: | :---: | :---: |
| 1 | 2500.00 | 1600 | 900 | 19100 |

$$
20000-900
$$

Flora borrows $\$ 20000$ at an interest rate of $8 \%$ per annum, compounding annually. She makes annual payments of $\$ 2500$.

Construct an amortisation table for Flora's reducing balance loan for the first three payments.

| Payment Num | Payment | Interest | Principal reduction | Balance |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\$ 0.00$ | $\$ 0.00$ | $\$ 0.00$ | $\$ 20,000.00$ |
| 1 | 2500 | 1600 | 900 | 19100.00 |
| 2 | 2500 | 1528 | 972 | 18128.00 |
| 3 | 2500 | 1450.24 | 1049.76 | 17078.24 |
| 4 | 2500 |  |  |  |

## Example: Amortisation tables

A business borrows $\$ 10000$ at a rate of $8 \%$ per annum, compounding quarterly. The loan is to be repaid by making quarterly payments of $\$ 2700.00$. The amortisation table for this loan is shown below.

$$
\frac{8}{4}=2 \%
$$

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 10000.00 |
| 1 | 2700.00 | 200 | 2500.00 | 7500.00 |
| 2 | 2700.00 | 150.00 | 2550 | 4950.00 |
| 3 | 2700.00 | 99.00 | 2601.00 | 2349 |

a Calculate the interest paid on the initial balance.
b Calculate the principal reduction from the second payment.
c Calculate the balance of the loan after payment 3 has been made.

## $2 \%$ of 10000

## Example: Amortisation tables

Consider the following amortisation table for an annuity after 3 monthly payments.

| Payment number | Payment | Interest | Principal reduction | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 12000.00 |
| 1 | 2200.00 | 60.00 | 2140.00 | 9860.00 |
| 2 | 2200.00 | 49.30 | 2150.70 | 7709.30 |
| 3 | 2200.00 | $38.55 A$ | $2161.45 B$ | 5547.85 |

a State the principal of the annuity and the amount of interest paid in the first month.
b Calculate the monthly interest rate.
c Find the value of $A$ and $B$.

$$
\$ 12000, \$ 60
$$



$$
\begin{aligned}
& A=38.55 \\
& B=2161.45
\end{aligned}
$$

## Amortisation tables for investments

These are pretty much the same as they were for loans and annuities however, here isn't a principal reduction there is a principal increase. The working out for all the other things is pretty much the same except:

Principal increase $=$ Payment + Interest
As, when we have an investment, the interest is added on to the payment.

| Payment number | Payment | Interest | Principal increase | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 1200.00 |
| 1 | 50.00 | 3.00 | 53.00 | 1253.00 |
| 2 | 50.00 | 3.13 | 53.13 | 1306.13 |
| 3 | 50.00 | 3.27 | 53.27 | 1359.40 |

## Example: Amortisation table for investments

Consider the following amortisation table for a compound interest investment with monthly additions to the principal. Assume that interest compounds monthly.

| Payment number | Payment | Interest | Principal increase | Balance |
| :---: | ---: | ---: | ---: | ---: |
| 0 | 0.00 | 0.00 | 0.00 | 1200.00 |
| 1 | 50.00 | 3.00 | 53.00 | 1253.00 |
| 2 | 50.00 | 3.13 | 53.13 | 1306.13 |
| 3 | 50.00 | 3.27 | 53.27 | 1359.40 |

Complete two additional lines for the table corresponding to payment 4 and payment 5 .
$0.25 \%$ of 1412.80

| Payment Num | Payment | Interest | Principal increase | Balance |
| :---: | :---: | :---: | ---: | :---: |
| 4 | 50 | 3.40 | 53.40 | 1412.80 |
| 5 | 50 |  |  |  |

## VCAA Questions

Pina invests $\$ 540000$ in an annuity paying $3 \%$ interest per annum, compounding monthly. Her annuity will provide a monthly payment of $\$ 5214.28$ for 10 years.
Four lines of the amortisation table for Pina's annuity are shown below.
The information for payment number 3 is missing.
a. What is the value of payment number 3 ?
b. Calculate the interest associated with payment number $3 . \quad V_{0}=540000, V_{n+1}=1.0025 V_{n}-5214.28$
Round your answer to the nearest cent.
c. Let $P_{n}$ be the balance, in dollars, of Pina's annuity after $n$ months.

Write a recurrence relation, in terms of $P_{0}, P_{n+1}$ and $P_{n}$, that can model this balance from month to month.


| Payment <br> number | Payment <br> (\$) | Interest <br> (\$) | Principal reduction <br> (\$) | Balance <br> (\$) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00 | 0.00 | 0.00 | 540000.00 |
| 1 | 5214.28 | 1350.00 | 3864.28 | 536135.72 |
| 2 | 5214.28 | 1340.34 | 3873.94 | 532261.78 |
| 3 | $\mathbf{5 2 1 4 . 2 8}$ | $\mathbf{1 3 3 0 . 6 5}$ | $\mathbf{3 8 8 3 . 6 3}$ |  |



## VCAA Questions

Deepa invests $\$ 500000$ in an annuity that provides an annual payment of $\$ 44970.55$

## VCAA 2021 Further Maths

Exam 1
Question 18
Interest is calculated annually.
The first five lines of the amortisation table are shown below.
The principal reduction associated with payment number 3 is
A. $\$ 17962.40$
B. $\$ 25969.37$
\$27008.15
D. $\$ 28088.47$
E. $\$ 44970.55$

| Payment number | Payment (\$) | Interest (\$) | Principal reduction (\$) | Balance (\$) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00 | 0.00 | 0.00 | 500000.00 |
| 1 | 44970.55 | 20000.00 | 24970.55 | 475029.45 |
| 2 | 44970.55 | 19001.18 | 25969.37 | 449060.08 |
| 3 | 44970.55 | 17962.40 | 27008.15 | 422051.93 |
| 4 | 44970.55 | 16882.08 | 28088.47 | 393963.46 |



## VCAA Questions

Samuel has a reducing balance loan.
The first five lines of the amortisation table for Samuel's loan are shown below.
Interest is calculated monthly and Samuel makes monthly payments of $\$ 1600$. Interest is charged on this loan at the rate of $3.6 \%$ per annum.

Exam
Question 8

$$
r=\frac{3.6}{12}=0.3 \%
$$

a. Using the values in the amortisation table
i. calculate the principal reduction associated with payment number 3
ii. calculate the balance of the loan after payment number 4 is made.

Round your answer to the nearest cent.


| Payment number | Payment <br> (\$) | Interest <br> (\$) | Principal reduction <br> (\$) | Balance <br> (\$) |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.00 | 0.00 | 0.00 | 320000.00 |
| 1 | 1600.00 | 960.00 | 640.00 | 319360.00 |
| 2 | 1600.00 | 958.08 | 641.92 | 318718.08 |
| 3 | 1600.00 | 956.15 | 643.85 | 318074.23 |
| 4 | 1600.00 | $\mathbf{9 5 4 . 2 2}$ | 645.78 | 317428.45 |



Making Maths
Easy, Engaging
Educational, Entertaining

Nevgstor: Heme


