Using recurrence relations to analyse and model reducing balance loans and annuities

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to model a reducing balance loan with a recurrence relation.
- To be able to use a recurrence relation to analyse a reducing balance loan.
- To be able to model an annuity with a recurrence relation.
- To be able to use a recurrence relation to analyse an annuity.

Recap

The last lesson in this series spent time recapping the work from the previous section.

As seems to be normal, we are using the following formula to apply recursion to finance applications:

$$V_0 = Principal, \qquad V_{n+1} = R \times V_n \pm D, \qquad where\ R = 1 \pm \frac{r}{100}$$

Reducing balance loans

When we take out a loan, the bank are lending us a (generally large) amount of money which we are agreeing to pay back over a period of time.

The amount loaned to us is called the **principal**.

The bank are going to charge us **interest** on the amount still owed. The interest will be quoted as a per annum (nominal) rate but will compound more regularly.

We are going to endeavour to pay back the money over a fixed period of time.

Note: Whilst short compounding periods are **really good** when we invest money, they are **not good** when we have a loan!!!

Annuity

Perpetuity

Interest only loans

30 years × 12 = 360

300 000 ↑ ↓

Reducing balance loans

When we have reducing balance loans we are going to use the following version of the formula:

$$V_0 = Principal, \qquad V_{n+1} = R \times V_n - D, \qquad where\ R = 1 + \frac{r}{100}$$

This is negative as we are paying money back to the bank and reducing the balance of what we still owe.

Example: Reducing balance loans

Flora borrows \$8000 at an interest rate of 13% per annum, compounding annually. She makes yearly payments of \$2100.

Construct a recurrence relation to model this loan, in the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

where V_n is the balance of the loan after n years.

$r = 13\%$

$R = 1 + \frac{13}{100}$

$= 1.13$

$V_0 = 8000, \quad V_{n+1} = 1.13 \cdot V_n - 2100$

Example: Reducing balance loans

Alyssa borrows $1000 at an interest rate of 15% per annum, compounding monthly. She makes monthly payments of $250.

Construct a recurrence relation to model this loan, in the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n - D$$

where V_n is the balance of the loan after n months.

$$r = \frac{15}{12}$$

$$R = 1 + \frac{1.25}{100}$$

$$R = 1.0125$$

$$V_0 = 1000, \quad V_{n+1} = 1.0125 . V_n - 250$$

Example: Reducing balance loans

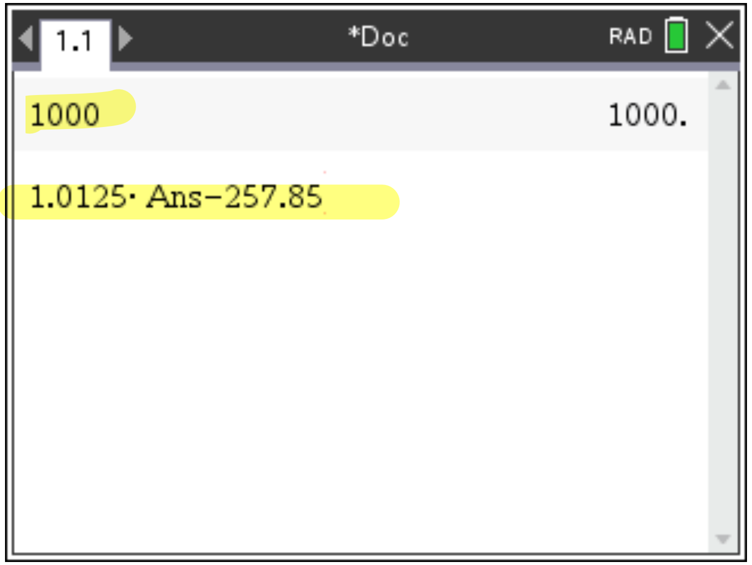
Alyssa's loan can be modelled by the recurrence relation:

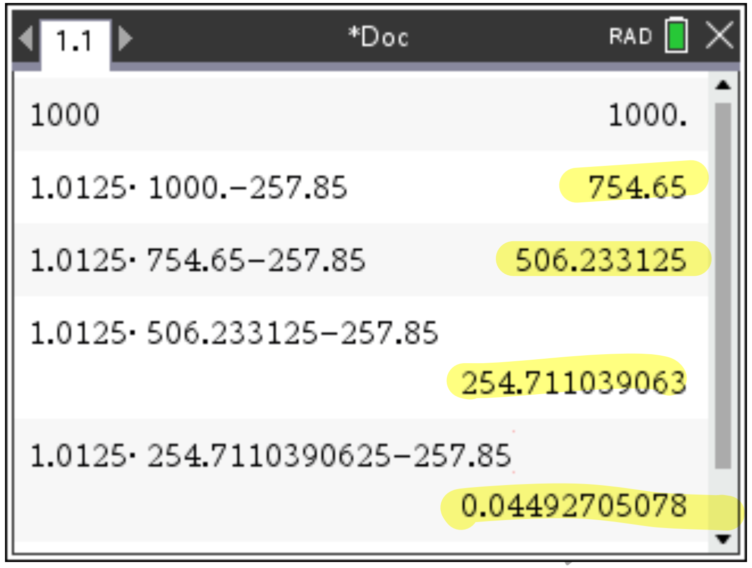
$$V_0 = 1000, \qquad V_{n+1} = 1.0125V_n - 257.85$$

a Use your calculator to find the balance of the loan after four payments. $0.04

b Find the balance of the loan after two payments have been made. Round your answer to the nearest cent.

$506.23





Annuities

When you start working (doing a real job and no Maccas!), you will probably want to pay into a Superannuation scheme. At the moment, in Victoria, business and companies should pay into your "Super" roughly 11% of your annual salary.

Super....

You can also opt to pay into your super each month to help the money grow more quickly.

This is designed to help you plan and pay for your retirement.

When you retire you are going to take a **fixed** amount of money from your account each month to help you live. You will still earn **compound interest** but, on the whole, the balance will go down each month.

This is called an Annuity.

100 000 ↑

\$1 000 000

Annuities: The equation

As you are earning interest, but withdrawing a fixed sum each month we can, once again, use the following formula:

$$V_0 = Principal, \qquad V_{n+1} = R \times V_n - D, \qquad where\ R = 1 + \frac{r}{100}$$

Note:
D will be the amount you withdraw each month
'r' will be the compounded interest rate per month etc.

Example: Annuities

Reza invests \$12 000 in an annuity that earns interest at the rate of 6% per annum, compounding monthly, providing him with a monthly income of \$2035.

a Model this annuity using a recurrence relation of the form

$$V_0 = \text{the principal}, \quad V_{n+1} = RV_n - D$$

where V_n is the value of the annuity after n months.

b Use your calculator to find the value of the annuity after the first four months. Round your answer to the nearest cent.

$r = \frac{6\%}{12}$

$= 0.5\%$

$R = 1 + \frac{0.5}{100}$

$R = 1.005$

$V_0 = 12000, \quad V_{n+1} = 1.005V_n - 2035$

Example: Annuities

Reza invests \$12 000 in an annuity that earns interest at the rate of 6% per annum, compounding monthly, providing him with a monthly income of \$2035.

a Model this annuity using a recurrence relation of the form

$$V_0 = \text{the principal}, \quad V_{n+1} = RV_n - D$$

where V_n is the value of the annuity after n months.

b Use your calculator to find the value of the annuity after the first four months. Round your answer to the nearest cent.

\$4040.55

$\frac{6}{12}$	$r =$	0.5
$\frac{0.5}{100}$		0.005
0.005+1	$R =$	1.005

0.005+1	1.005
12000	12000.
1.005· 12000.−2035	10025.
1.005· 10025.−2035	8040.125
1.005· 8040.125−2035	6045.325625
1.005· 6045.325625−2035	4040.55225313

VCAA Questions

VCAA 2022 Further Maths
Exam 1

Use the following information to answer Questions 18 and 19.

The balance of a loan, V_n, in dollars, after n months is modelled by the recurrence relation

$$V_0 = 400\,000, \quad V_{n+1} = 1.003V_n - 2024$$

Question 18

The balance of the loan first falls below \$398 000 after how many months?

A. 1
B. 2
C. 3
D. 4
E. 5

VCAA Questions

VCAA 2022 Further Maths
Exam 2

Question 8 (4 marks)

To purchase additional workplace equipment, Pina took out a reducing balance loan of \$580 000 with interest calculated monthly.

The balance of the loan, in dollars, after n months, L_n, can be modelled by the recurrence relation

$$L_0 = 580\,000, \qquad L_{n+1} = 1.002L_n - 3045.26$$

a. Showing recursive calculations, determine the balance of the loan after two months.
Round your answer to the nearest cent. 1 mark

$L_0 = 580\,000$

$L_1 = 1.002 \times 580\,000 - 3045.26 = 578\,114.74$

$L_2 = 1.002 \times 578\,114.74 - 3045.26 = \$576\,225.71$

Question 8 (4 marks)

To purchase additional workplace equipment, Pina took out a reducing balance loan of $580 000 with interest calculated monthly.

The balance of the loan, in dollars, after n months, L_n, can be modelled by the recurrence relation

$$L_0 = 580\,000, \qquad L_{n+1} = 1.002L_n - 3045.26$$

b. Determine the annual compound interest rate for this loan. 1 mark

$R = 1.002$

$R = 1 + \frac{r}{100}$

$1.002 = 1 + \frac{r}{100}$

$r = 0.2\% \times 12$

$\therefore$ Annual rate $= \underline{\underline{2.4\%}}$

VCAA Questions

VCAA 2021 Further Maths
Exam 2

Question 8 (3 marks)

For renovations to the coffee shop, Sienna took out a reducing balance loan of \$570 000 with interest calculated fortnightly.

The balance of the loan, in dollars, after n fortnights, S_n, can be modelled by the recurrence relation

$$S_0 = 570\,000, \qquad S_{n+1} = 1.001S_n - 1193$$

a. Calculate the balance of this loan after the first fortnightly repayment is made. 1 mark

\$569 377

VCAA 2021 Further Maths
Exam 2

Question 8 (3 marks)

For renovations to the coffee shop, Sienna took out a reducing balance loan of \$570 000 with interest calculated fortnightly.

The balance of the loan, in dollars, after n fortnights, S_n, can be modelled by the recurrence relation

$$S_0 = 570\,000, \qquad S_{n+1} = 1.001S_n - 1193$$

b. Show that the compound interest rate for this loan is 2.6% per annum. 1 mark

$R = 1.001$

$1.001 = 1 + \frac{r}{100}$

$r = 0.1\%$

p.a $= 0.1 \times 26$

$= \underline{\underline{2.6\%}}$

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