Perfect squares and difference of perfect squares

Year 9 Mathematics
Mainstream

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Learning Objectives

By the end of the lesson I hope that you understand and can apply the following to a range of questions from the Year 9 Mathematics course.

- To be able to identify a perfect square
- To be able to expand a perfect square
- To understand what type of expansion forms a difference of perfect squares (DOPS)
- To be able to expand to form a difference of perfect squares

Recap of past learning

In the last lesson we looked at expanding and simplify binomial products.

We learned that we can use FOIL to help us alongside our understanding of collecting like terms. We even learned how we might look at patterns and how each part of the expansion was created.

Building on this, we are going to look at certain types of binomial expansion and how they can create things called "perfect squares".

$\times$	x	$+3$
x	x^2	$+3x$
$+2$	$+2x$	$+6$

$$(x+3)(x+2)$$

$$= x^2 \underbrace{+3x+2x}+6$$

$$x^2+5x+6$$

What is a perfect square?

A square who doesn't do anything wrong?

Probably … but in Mathematics it is anything which can be raised to a power of 2.

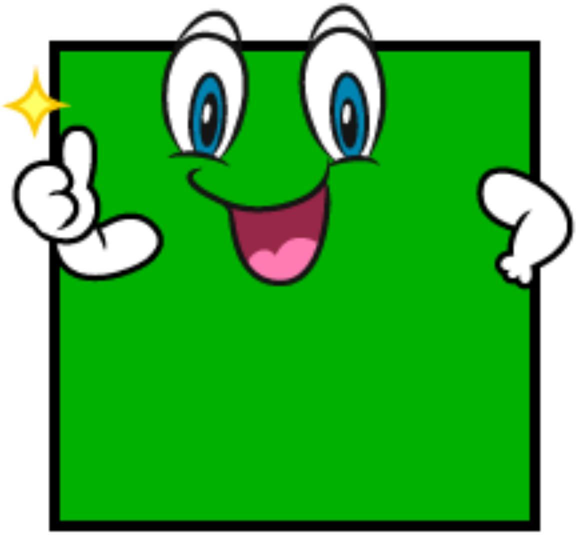
Floaty numbers here we come!!!

Examples:

$$3^2 \ or \ a^2 \ or \ (3x)^2 \ or \ (x+2)^2$$

Hold on, we have met something in brackets like this before!

When you **square something you multiply by itself.**



$9x^2 = (3x)^2$

$16x^2 = (4x)^2$

$(x+2)^2 = (x+2)(x+2)$

Lots of people stuff this up

Let's be the first group to never make this common mistake.

When we square a set of brackets, we need to write it out in long notation **before we multiply it**.

For example:

$$(x+2)^2 = (x+2)(x+2)$$

We have learned how to multiply this out in the last lesson using **FOIL**

Remember:
First
Inside
Outside
Last

$$x^2 + 2x + 2x + 4$$
$$= x^2 + 4x + 4$$

Examples

Expand each of the following.

a $(x-2)^2$

b $(2x+3)^2$

$$(x-2)^2 = (x-2)(x-2)$$

$$= x^2 - 2x - 2x + 4$$

$$= x^2 - \underline{4x} + 4$$

$$(2x+3)^2 = (2x+3)(2x+3)$$

	$2x$	$+3$
$2x$	$4x^2$	$+6x$
$+3$	$+6x$	$+9$

$$4x^2 + 6x + 6x + 9$$

$$= 4x^2 + \underline{12x} + 9$$

Do we always get three terms (once we have simplified)?

Pretty much all of the expansions we have done so far have resulted in us ended up with three terms:

$$(x+2)(x+3) = x^2 + 5x + 6$$
$$(x-2)(x+3) = x^2 + x - 6$$
$$(2x+3)(x-2) = 2x^2 - x - 6$$

What happens with the following examples:

$$(x+2)(x-2)$$
$$(x+3)(x-3)$$
$$(2x-4)(2x+4)$$

$$(x+2)(x-2) = x^2 \cancel{-2x+2x} - 4$$
$$x^2 - 4$$

$$(x+3)(x-3) = x^2 \cancel{-3x+3x} - 9$$
$$= x^2 - 9$$

What do you notice?

Do you notice anything about the structure of the brackets?

$$(x+2)(x-2)$$
$$(x+3)(x-3)$$
$$(2x-4)(2x+4)$$

What is the same? What is different?

And what about the answers?

$$(x+2)(x-2)=x^2-4$$

$$(x+3)(x-3)=x^2-9$$

$$(2x-4)(2x+4)=4x^2-16$$

$$(2x)^2-(4)^2$$

1
4
9
16
25
36
49
64

Difference of perfect squares

You will notice that the answers are what we call perfect squares:

$$(x+2)(x-2) = x^2 - 4$$
$$= (x)^2 - (2)^2$$

$$x^2 - 25$$
$$= x^2 - (5)^2$$
$$= (x+5)(x-5)$$

$$(x+3)(x-3) = x^2 - 9$$
$$= (x)^2 - (3)^2$$

Only works when there is a negative between two squares

Examples

Expand and simplify the following.

a $(x + 2)(x - 2)$

b $(3x - 2y)(3x + 2y)$

$3x^2$

$(x+2)(x-2)$

$x^2 - 2^2 = x^2 - 4$

$(3x - 2y)(3x + 2y) = (3x)^2 - (2y)^2$

$= 9x^2 - 4y^2$

Examples

Expand and simplify the following.

a $(x+4)(x-4)$ **b** $(5x-3y)(5x+3y)$

$$(x+4)(x-4) = (x)^2 - (4)^2$$
$$= x^2 - 16$$

$$(5x-3y)(5x+3y) = (5x)^2 - (3y)^2$$
$$= 25x^2 - 9y^2$$

Examples

Expand the following.

a $(x + 3)(x + 5)$ **b** $(x - 4)(x + 7)$ **c** $(2x - 1)(x - 6)$ **d** $(5x - 2)(3x + 7)$

$(x + 3)(x + 5) =$

$= x^2 + 5x + 3x + 15$

$= x^2 + \underline{8x} + 15$

Remember:
First
Inside
Outside
Last

Note:
Be careful of changing signs!

Questions to complete

The following work is the **minimum** you are expected to complete in class and at home.

You are welcome to answer more questions if you feel you have the time.

Exercise 8B
Questions: 2aei, 3ajm, 4adg, 5adj, 6abcde, 7, 8,

Extension: 12abcd