Combining linear and geometric growth or decay to model compound interest investments with additions to the principal

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to generate a sequence from a recurrence relation that combines both geometric and linear growth or decay.
- To be able to model compound interest investments with additions to the principal.
- To be able to use a recurrence relation to analyse compound interest investments with additions to the principal.
- To be able to determine the annual interest rate from a recurrence relation.

Recap

This is a new section of the course but builds heavily on the work we have done in the previous section relating to recurrence relations and rules. If you haven't already completed the section I would suggest going back and taking a look.

This chapter is generally a recap of the work from the previous chapter.

Note: A CAS calculator is going to be vital for the success in this section.

It is vital that you know how to use the CAS and the finance solver to gain the maximum of marks.

Recap: Generating a sequence from a recurrence relation

We know a sequence is a list of numbers starting from a number and the creation of which will follow some sort of rule. For example:

$$V_0 = starting\ number, \qquad V_{n+1} = R \times V_n \pm D$$

When the value of R is one, we will have a linear sequence.

When the value of R is between 0 and 1 we will have a geometric sequence (so long as there is no value of D).

Example: Generating a sequence from a recurrence relation

Write down the first five terms of the sequence generated by the recurrence relation

$$V_0 = 3, \qquad V_{n+1} = 4V_n - 1$$

3 11 43 171 683

Compound interest investments with a regular addition to principal

In the previous chapter we didn't have any examples where we had a value for both R and D. We generally used the following equation

$$V_0 = starting\ number, \qquad V_{n+1} = R \times V_n$$

Where the value of R was calculated using:

$$R = 1 \pm \frac{r}{100}$$

Where we used either the + or the – depending on the context of the question.

Now we are going to look at examples where we have both an R value and a D value.

Recap: Different compounding periods

Interest rates are normally given as a per annum (per year) figures.

We learned, in the previous section of the course, that we can actually compound interest in lots of different ways.

The more common are quarterly, monthly, fortnightly, weekly and daily.

Being able to covert a per annum (nominal) interest rate to other rates is really important.

Note: The context of the question will tell you which interest rate to use.

9% p.a.

$\frac{9}{12}\%$

RTFQ.

Example

Fred has saved \$5000 and invests this in a compound interest account paying 4% per annum, compounding yearly. He also adds an extra \$1000 each year.

Model this investment using a recurrence relation of the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n + D$$

where V_n is the value of the investment after n years.

$r = 4\%$

$R = 1 + \frac{4}{100}$

$= 1.04$

$V_0 = 5000, \quad V_{n+1} = 1.04 \times V_n + 1000$

Example

Nor invests \$1200 and plans to add an extra \$50 each month. The account pays interest at a rate of 3% per annum, compounding monthly.

Model this investment using a recurrence relation of the form

$$V_0 = \text{the principal}, \qquad V_{n+1} = RV_n + D$$

where V_n is the value of the investment after n months.

$r = \frac{3}{12} = 0.25\%$

$R = 1 + \frac{0.25}{100} = 1.0025$

$V_0 = 1200, \quad V_{n+1} = 1.0025\,V_n + 50$

Example

Albert has an investment that can be modelled by the recurrence relation

$$V_0 = 400, \qquad V_{n+1} = 1.005V_n + 30$$

where V_n is the value of the investment after n months.

a State the value of the initial investment. $400

b Determine the value of the investment after Albert has made three extra payments. Round your answer to the nearest cent. $496.48

c What will be the value of his investment after 6 months? Round your answer to the nearest cent. $594.42

d Plot the points for the value of the investment after 0, 1, 2 and 3 months on a graph.

Example

Albert has an investment that can be modelled by the recurrence relation

$$V_0 = 400, \qquad V_{n+1} = 1.005V_n + 30$$

where V_n is the value of the investment after n months.

a State the value of the initial investment.

b Determine the value of the investment after Albert has made three extra payments. Round your answer to the nearest cent.

c What will be the value of his investment after 6 months? Round your answer to the nearest cent.

d Plot the points for the value of the investment after 0, 1, 2 and 3 months on a graph.

Example

Determine the annual interest rates for each of the following investments.

a Consider an investment given by the recurrence relation

$$A_0 = 400, \qquad A_{n+1} = 1.005V_n + 30$$

where A_n is the value of the investment after n months.

b Consider an investment given by the recurrence relation

$$W_0 = 2000, \qquad W_{n+1} = 1.012V_n + 500$$

where W_n is the value of the investment after n quarters.

$R = 1.005$

$R = 1 + \frac{r}{100}$

$\text{Solve}\left(1.005 = 1 + \frac{r}{100}, r\right)$

$r = 0.5\%$

$\times 12$

$= 6\%$

Example

Determine the annual interest rates for each of the following investments.

a Consider an investment given by the recurrence relation

$$A_0 = 400, \qquad A_{n+1} = 1.005V_n + 30$$

where A_n is the value of the investment after n months.

b Consider an investment given by the recurrence relation

$$W_0 = 2000, \qquad W_{n+1} = 1.012V_n + 500$$

where W_n is the value of the investment after n quarters.

$R = 1.012$

$R = 1 + \frac{r}{100}$

$1.012 = 1 + \frac{r}{100}$

$r = 1.2\%$

$\times 4$

$r = 4.8\%$

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