# Operations with Fractions

Sunday, 12 April 2020 10:48 am

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- How to add, subtract, multiply and divide fractions
- We will recap how to turn whole numbers into fractions, cross cancelling, cancelling down, mixed numbers and how to find the common denominator

## RECAP

This is the first lesson in a series dealing with Real Numbers.
Much of the work has already been covered in Year 7 and, as such, this will be more of a review of how to perform each of the operations with Fractions.
It cannot be stated firmly enough that fractions play a very important part of Mathematics for the rest of the course. Whilst Fractions are not loved by anyone, you must continue to practice, practice and practice how to do the basic operations. Once you have mastered this, you will be able to do a lot of Mathematics with relative ease.

## The four operations

There are 4 operations which you need to understand how to apply with fractions:

- Adding
- Subtracting
- Multiplying, and
- Dividing

The easiest way to teach fractions is to do them in the following order:

- Multiplying
- Dividing
- Adding, then
- Subtracting

## RECAP: Whole Numbers are fractions

It is important to know how to turn a whole number into a fraction.

**Short cut**: Each whole number can be divided by one to turn it into a fraction

$12 = \frac{12}{1}$

$16 = \frac{16}{1}$

$14 = \frac{14}{1}$

$1234 = \frac{1234}{1}$

$6.7 = \frac{6.7}{1}$ ×

## RECAP: Cancelling down fractions

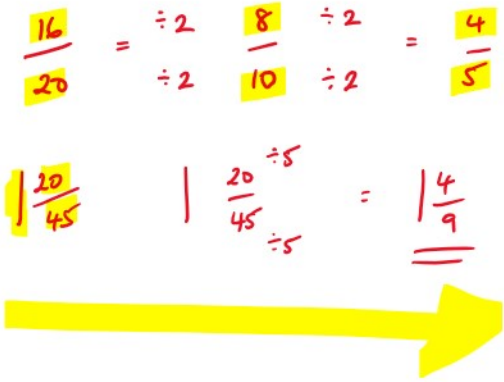
You must know your times tables to be able to effectively cancel down fractions.
If you don't think you know them, then it's vital that you learn them.

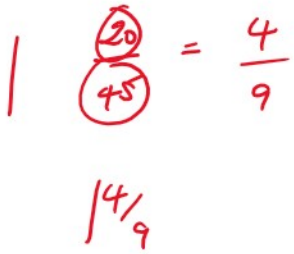
You will lose all marks in a question if you don't, with a "final" answer, cancel the fraction down.

**Short cut**: Keep dividing the numerator and denominator by the same number until they cannot be divided any more. You may need to keep changing the number you divide by.

$\frac{12}{15} \overset{\div 3}{\underset{\div 3}{=}} \frac{4}{5} \overset{\div 1}{\underset{\div 1}{=}} \frac{4}{5}$

$\frac{16}{20} \overset{\div 2}{\underset{\div 2}{=}} \frac{8}{10} \overset{\div 2}{\underset{\div 2}{=}} \frac{4}{5}$

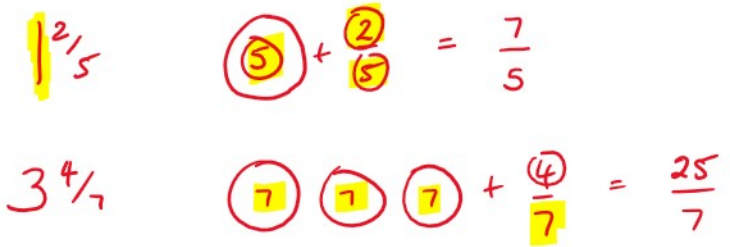


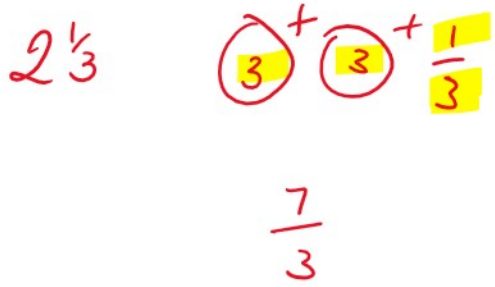


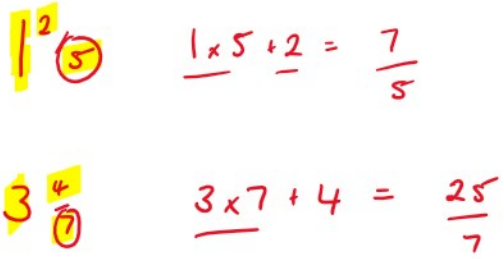
## RECAP: Mixed numbers

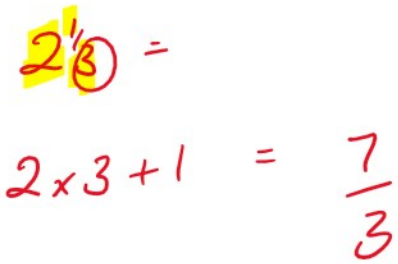
It is easier to convert mixed numbers into top heavy fractions to be able to make mathematical processes easier.

**Short cut**: Multiply the whole number by the denominator and then add the numerator. Ensure you divide this number by the original denominator.







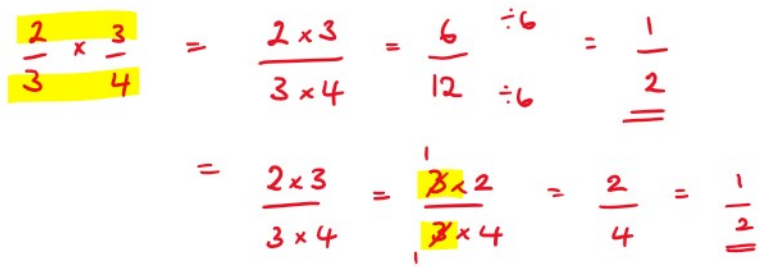


## Multiplying fractions

This is considered the easiest operation to perform on fractions.

Note: It doesn't matter what the denominator of a fraction is for this process to work.

**Short cut**: You multiply the two top numbers together and multiply the two bottom numbers together.



$3 \times 4 \quad 3 \times 4 \quad 4 =$

$\frac{4}{7} \times \frac{3}{5} = \frac{4 \times 3}{7 \times 5} = \frac{12}{35}$

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1 \times 2 \times 3}{2 \times 3 \times 4} = \frac{6 \div 6}{24 \div 6} = \frac{1}{4}$

$= \frac{1 \times 2 \times 1}{2 \times 3 \times 4} = \frac{1}{4}$

$1\frac{1}{2} \times \frac{2}{3} = \frac{3}{2} \times \frac{2}{3} = \frac{6}{6} = 1$

$2\frac{2}{5} \times \frac{1}{3} \qquad 2 \times 5 + 2 \qquad \frac{12}{5} \times \frac{1}{3} = \frac{12}{15} = \frac{4}{5}$ (÷3)

## Dividing fractions

This is considered the second easiest operation to perform on fractions.
It builds on the process of multiplying the fractions by adding one more step.

**Short cut**: Keep -> Change -> Flip and the multiply as normal

Note: Flipping a fraction is also called "taking the reciprocal"

KCF

$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{1 \times 3}{2 \times 2} = \frac{3}{4}$

KEEP CHANGE FLIP

$\frac{1}{2} \div \frac{2}{5}$

K C F

$\frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$

$= 5 \div 4$

$\frac{7}{9} \div \frac{1}{3} = \frac{7}{9} \times \frac{3}{1} = \frac{7 \times 3}{9 \times 1} = \frac{21}{9} = \frac{7}{3}$

Whole ← new

$1\frac{1}{4}$

$2\frac{2}{5} \div 1\frac{1}{2} = \frac{12}{5} \div \frac{3}{2} = \frac{12}{5} \times \frac{2}{3} = \frac{24}{15} = \frac{8}{5}$

$= 1\frac{3}{5}$

## Adding fractions

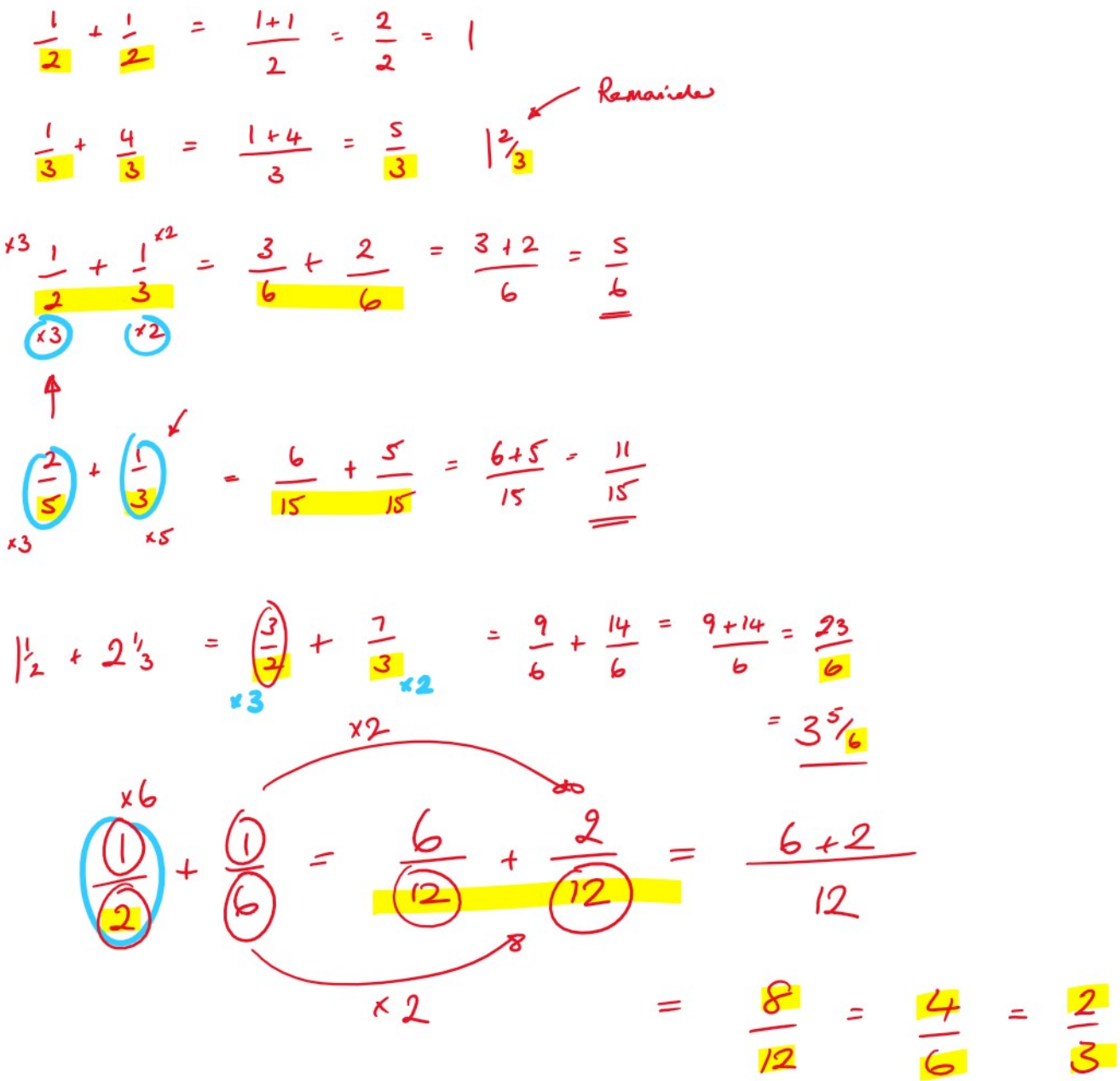
This is considered more complex than the previous two methods.
The key fact is that **you can only add or subtract fractions if the denominators are the same.**
We make the two fractions have a **common denominator**.

Once the denominators are the same you can then add them.

**Short cut:** Add the numerators. Leave the denominator the same.

### RECAP: Common denominators

The easier way to make the denominators the same is to follow this process




### Subtracting fractions

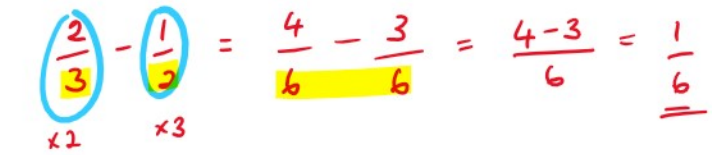
This uses the same process as adding fractions but you are subtracting them.
The key fact is that **you can only add or subtract fractions if the denominators are the same.**
We make the two fractions have a **common denominator**.

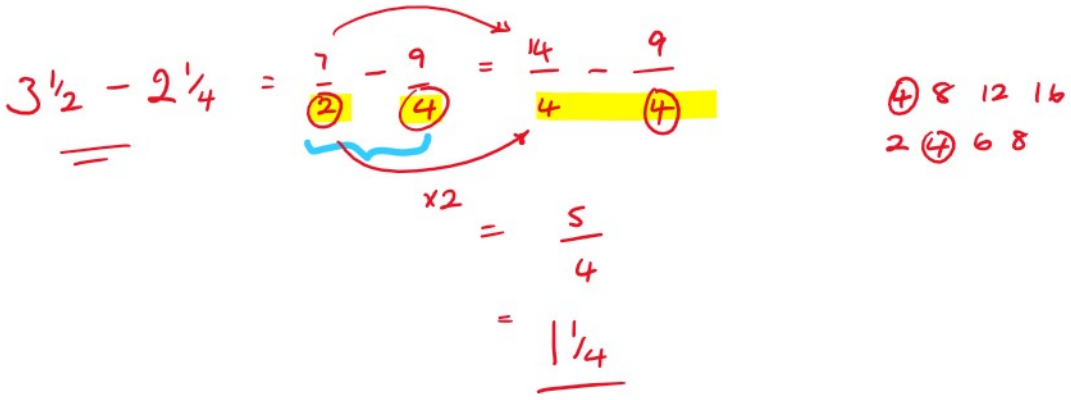
Once the denominators are the same you can then subtract them.

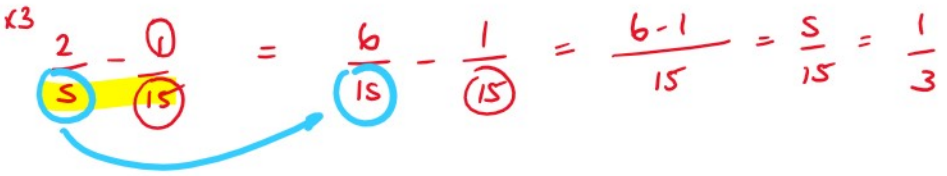
**Short cut:** Subtract the numerators. Leave the denominator the same.

Note: You must cancel down if needed.

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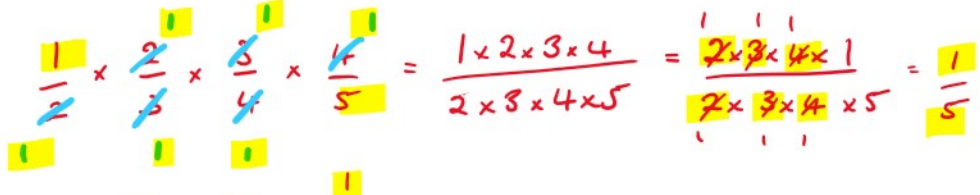


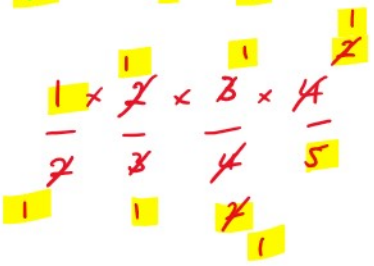


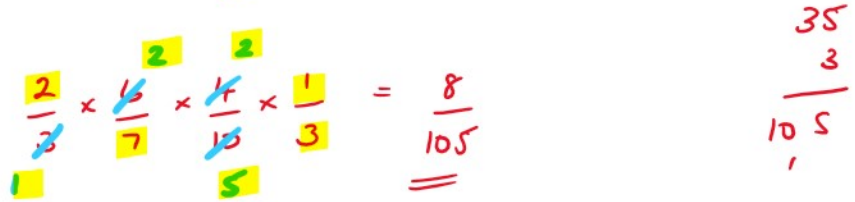
## Cross cancelling when multiplying fractions

There is an awesome process called cross cancelling which can be used to make life easier.
By cancelling at the start of the question we can remove the need to simplify at the end of the question.

Note: This only works for **multiplying fractions**.







**Examples**

The following examples have been extracted, with permission, from the Cambridge Essentials Series of textbooks (Year 8 in this case).

Simplify the following

$\frac{3}{5}+\frac{4}{5}$ $= \frac{3+4}{5} = \frac{7}{5}$ $1\frac{2}{5}$

$\frac{5}{3}-\frac{3}{4}$ $\frac{5}{3} - \frac{3}{4} = \frac{20}{12} - \frac{9}{12}$

$\times 4 \quad \times 3$

$= \frac{20-9}{12} = \frac{11}{12}$

$3\frac{5}{8}+2\frac{3}{4}$ $= 3\frac{5}{8} + 2\frac{3}{4}$

$= \frac{29}{8} + \frac{11}{4} = \frac{29}{8} + \frac{22}{8} = \frac{29+22}{8}$

$\times 2$

$= \frac{51}{8} = 6\frac{3}{8}$

$2\frac{1}{2}-1\frac{5}{6}$ $= 2\frac{1}{2} - 1\frac{5}{6} = \frac{5}{2} - \frac{11}{6} = \frac{15}{6} - \frac{11}{6}$

$\times 3$

$= \frac{15-11}{6}$

$= \frac{4}{6} = \frac{2}{3}$

$\frac{2}{5}\times\frac{3}{7}$ $= \frac{2\times 3}{5\times 7} = \frac{6}{35}$

$\frac{2}{5} \times \frac{3}{7}$ $= \frac{2 \times 3}{5 \times 7} = \frac{6}{35}$

We will do the following example in two ways; the hard way and cross cancelling.

$\frac{8}{5} \times \frac{7}{4}$

① $\frac{8}{5} \times \frac{7}{4} = \frac{8 \times 7}{5 \times 4} = \frac{56}{20} = \frac{28}{10} = \frac{14}{5} = 2\frac{4}{5}$

② $\frac{\overset{2}{\cancel{8}}}{5} \times \frac{7}{\cancel{4}_{1}} = \frac{2 \times 7}{5 \times 1} = \frac{14}{5} = 2\frac{4}{5}$

$3\frac{1}{3} \times 2\frac{2}{5}$ $= 3\frac{1}{3} \times 2\frac{2}{5}$

$= \frac{\overset{2}{\cancel{10}}}{\cancel{3}_{1}} \times \frac{\cancel{12}^{4}}{\cancel{5}_{1}}$

$= \frac{2 \times 4}{1 \times 1} = \frac{8}{1} = 8$

$\frac{2}{5} \div \frac{3}{7}$ $= \frac{2}{5} \div \frac{3}{7} = \frac{2}{5} \times \frac{7}{3} = \frac{2 \times 7}{5 \times 3}$

$= \frac{14}{15}$

$\frac{14}{15}$

$\frac{5}{8} \div \frac{15}{16}$ $= \frac{\cancel{5}^{1}}{\cancel{8}_{1}} \times \frac{\cancel{16}^{2}}{\cancel{15}_{3}} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$

$2\frac{1}{4} \div 1\frac{1}{3}$ $= 2\frac{1}{4} \div 1\frac{1}{3}$

$= \frac{9}{4} \div \frac{4}{3}$

$= \frac{9}{4} \times \frac{3}{4}$

$= \frac{27}{16} = 1\frac{11}{16}$