Area of sectors

Thursday, 7 May 2020 9:03 am

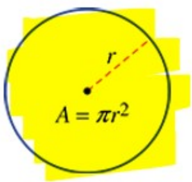
By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

- Understand what a sector is
- Understand that a sector is effectively a fraction of a circle
- Understand how to find the area of a sector using the radius (or diameter) and angle
- Find areas of composite shapes

RECAP

In the previous lesson we looked at how to find the area of a complete circle using the following formula:

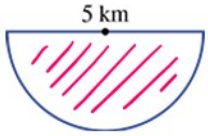
$$Area = \pi r^2$$



$A = \pi \times r^2 \qquad r^2 = r \times r$

$A = \pi \times r \times r$

We need to remember that this gave us the area of the **whole circle**.
Questions were sent to try and trick us:



$A_{\circ} = \pi \times r \times r$

$= \pi \times 2.5 \times 2.5$

$A_{\smile} =$

In fact, the trick was not completely understanding how we can extend the answer from the question above into a more generic formula.

RECAP: What is a fraction

Remember, a fraction is nothing more than a section of a whole.
A slice of pizza is a fraction of a whole pizza.
The fraction depends on how many slices a pizza can be cut into

$\frac{1}{6} \qquad \frac{4}{6}$

RECAP: Degrees in a circle

Remember that there are 360° in a circle.
So, we can think of a circle as being a pizza which is cut into 360 slices.

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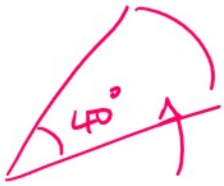


$\frac{40}{360}$

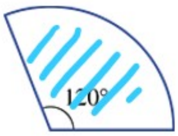
$\frac{1}{2}$ $\quad \frac{1}{2}\pi r^2$

$\frac{1}{4}$ $\quad \frac{1}{4}\pi r^2$

$A = \frac{40}{360} \times \pi r^2$

Using slices to find a general formula for the area of a sector

When we cut a circle into sections, then we are really finding the fraction of the whole circle.



$\frac{90}{360} = \frac{1}{4}$ $\qquad \frac{60}{360} = \frac{1}{6}$ $\qquad \frac{120}{360} = \frac{1}{3}$

If we know the fraction of the circle we have, then we can find the area of the fraction of the circle.

We can find the area of the whole circle and then multiply it by the fraction.

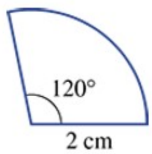
This can be more simply put as:

$$Area\ of\ sector = \frac{(Angle\ of\ sector)}{360} \times \pi r^2$$

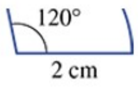
Let's do some examples

The following examples are taken, with permission, from the Cambridge Essentials Year 8 Textbook

Find the area of these sectors correct to two decimal places.



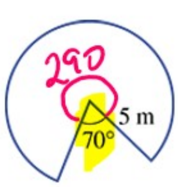
$A = \frac{120}{360} \times \pi r^2$



$$360$$

$$= \frac{120}{360} \times \pi \times 2^2$$

$$= \underline{4 \cdot 19 \text{ cm}^2}$$



$$\begin{array}{r} 360^\circ \\ -\ 70 \\ \hline 290 \end{array}$$

$$A = \frac{290}{360} \times \pi \times 5^2$$

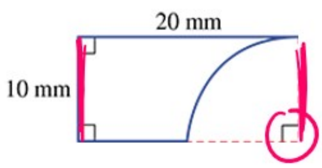
$$= \underline{63 \cdot 27 \text{ m}^2}$$

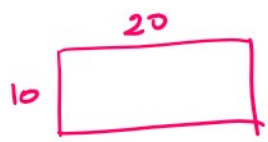
Theris nothing more complicate than that!

Well, until we meet composite shapes!

Examples of composite shapes
The following examples are taken, with permission, from the Cambridge Essentials Year 8 Textbook

Find the area of this composite shape correct to the nearest whole number of mm^2.



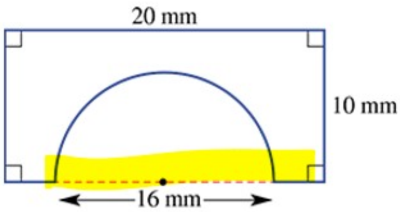


$$A_{\square} = 10 \times 20 = 200 \text{ mm}^2$$

$$A_{D} = \underline{90} \times \pi \times 10^2 = 78 \cdot 5398 \ldots \text{mm}^2$$

$$A = \frac{90}{360} \times \pi \times 10^2 = 78.5398\ldots \text{ mm}^2$$

$$A = 200 - 78.5398\ldots = \underline{121.46 \text{ mm}^2}$$



$$A_{\square} = 20 \times 10$$
$$= \underline{200 \text{ mm}^2}$$

$$A = \frac{180}{360} \times \pi \times 8^2$$
$$= 100.53\ldots \text{ mm}^2$$

$$\therefore A = 200 - 100.53\ldots$$
$$= \underline{99.47 \text{ mm}^2}$$