Area of a circle

Sunday, 12 April 2020 10:49 am

★ By the end of the lesson I would hope that you have an understanding and be able to apply to questions the following concepts:

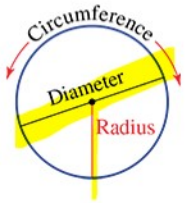
- Understand how to find the area of a circle using the radius (or diameter)
- Find areas of parts of circles
- Find areas of circles using approximations.

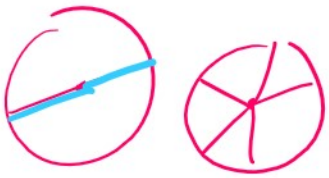
RECAP

This is the second lesson in this series on Real Numbers looking at Circles.
In the last lesson we looked at finding the circumference of a circle.
We learned that the circumference can be calculated using:

$D = 2 \times r$

$$Circumference = 2 \times \pi \times Radius$$



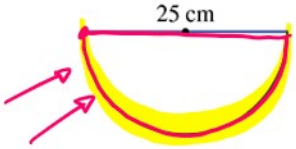


This seems easy on the surface, but it soon became tricky.

This is where it was easy for me to see who **understood the work** and who was simply **regurgitating content**.

Using a formula isn't Maths sadly. Understanding what the formula means and how to use it is vital for long term success in Mathematics.

For example, the following questions caused some concern.



$C = 2\pi r$

C_0

$C_0 \div 2$

$+ 25$

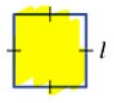


But it was a trick.

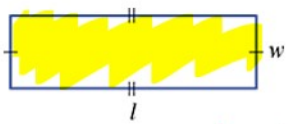
The same trick is going to be used in today's lesson!

The space inside

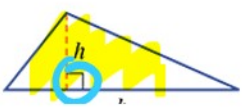
The space inside a 2D shape is called the area.
You are used to working out areas of many different shapes.

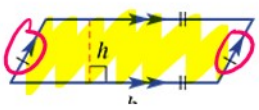


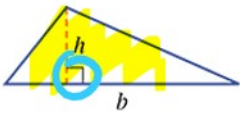
$A = l \times l$



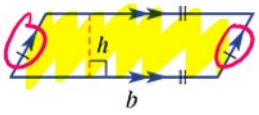
$A = l \times w$







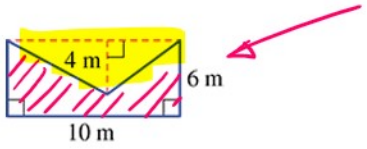
$A = \frac{1}{2} \times b \times h$



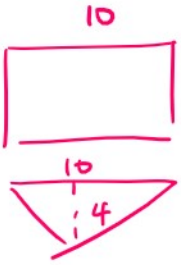
$A = b \times h$



We then moved onto combine basic shapes into what we call composite shapes.
There are lots of composite shapes.
An example is shown below:



$A = 60 - 20$
$= 40 m^2$



$= 60 m^2$

$= \frac{1}{2} \times 10 \times 4 = \frac{1}{2} \wedge 40 = 20 m^2$

The space inside a Circle

The area of a full circle is once again found using a formula.
Strangely, this is the one which most people seem to remember.
There are lots of ways to express the same formula.

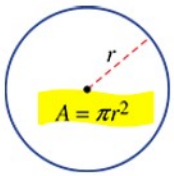
$The\ area\ of\ a\ circle = \pi \times (radius)^2$

$2^2 = 2 \times 2$

$3^2 = 3 \times 3$

$5^2 = 5 \times 5$

$r \times r$
$= r^2$



$Area = \pi r^2$

$Area = \pi \times r \times r$

$Area = pi \times radius \times radius$

All the above are exactly the same.

The most common one is:

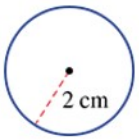
$$Area = \pi r^2$$

Hence, to find the area you simply need the radius. Which is strangely the same as we needed for the circumference.

Finding the area inside a circle (some examples)

Examples
The following examples are taken, with permission, from the Cambridge Essentials Year 8 Textbook

Use a calculator to find the area of this circle correct to two decimal places.

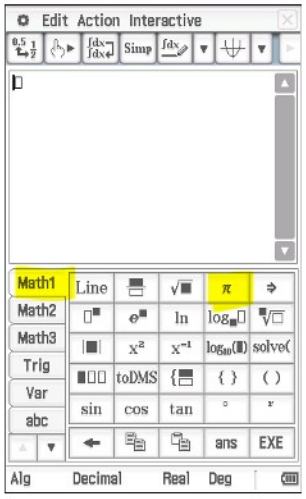


$A = \pi \times r^2$

$A = \pi \times 2^2$

$A = 12.57 \text{ cm}^2$

Notice that we're allowed to use a calculator in this question. Hence, we're going to use the π button.



There will be questions where you have to **approximate** the area and use different values of π

Remember:

$\pi \approx 3$

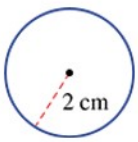
$\pi \approx 3.14$

$\pi \approx 3.142$

$\pi \approx \frac{22}{7}$

The question will let you know which approximation to use.

Let's find the area of the question then:



$\pi = 3$

$A = \pi \times r^2$

$A = 3 \times 2^2$

$A = 3 \times 4$

$A = 12 \text{ cm}^2$

$3.14 \times 4 = 12.56$

$\pi = 3.14$

$A = \pi \times r^2$

$= 3.14 \times 2^2$

$\pi = 3.14$

$A = \pi \times r$

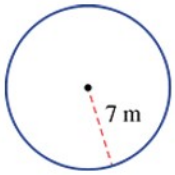
$= 3.14 \times 2^2$

$= 3.14 \times 4$

$= 12.56 \text{ cm}^2$

Another example:

Find the area of these circles using the given approximate value of π.



Use $\pi = \frac{22}{7}$

$A = \pi \times r^2$

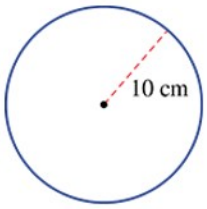
$= \frac{22}{7} \times 7^2$

$= \frac{22}{7} \times \frac{49}{1}^{7}$

$= 22 \times 7$

$= 154 \text{ m}^2$

$22 \times 7 = 154$



Use $\pi = 3.14$

$A = \pi \times r^2$

$= 3.14 \times 10^2$

$= 3.14 \times 100$

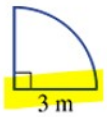
$= 314 \text{ cm}^2$

And now back to the tricks!

Remember, the **area for a whole circle is:**

$$Area = \pi r^2$$

How would I use the above to find the area of the following? Use the π button to find the answers.



$\frac{90°}{360°} = \frac{1}{4}$

$A_D = 28.27\ m^3$

$A_C = \pi \times r^2$

$= \pi \times 3^2$

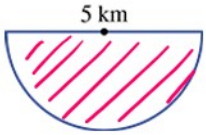
$= \pi \times 9$

$A_D = \frac{1}{4} \times A_C$

$= \frac{1}{4} \times 28.274\ldots$

$= 7.07\ m^2$

Find the area of the following. Don't be tricked by the same trick for circumference!



$D = 5$

$r = 2.5\ km$

$A_C = \pi \times r^2$

$= \pi \times 2.5^2$

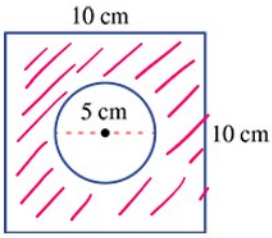
$= 19.63\ km^2$

$A_D = \frac{1}{2} \times 19.6349\ldots$

$= 9.82\ km^2$

One of the most popular type of exam questions

Remember, that we can now use circles in our composite shapes.
How would we find the area of the following shaded shape?





$A_C = \pi \times 2.5^2$

$= 19.634\ cm^2$

$A_S = 10 \times 10$

$= 100\ cm^2$

$A_{\text{shaded}} = 100 - 19.6349\ldots$

$= 80.37\ cm^2$

$= 80.37\ cm^2$

Doing it backwards!

How would we find the **radius** if given the area?

We have to be able to do things backwards.

We can use the solve function on a calculator, or the algebra we have been taught in previous modules.

For example:

Find the radius of a circle with an Area of 50 cm^2

$A = \pi \times r^2$

$\div \pi$ $\quad 50 = \pi \times r^2$ $\quad \div \pi$

$\frac{50}{\pi} = r^2$

$\sqrt{\frac{50}{\pi}} = r$

$r = 3.99\ cm$

$r = 3.99\ cm$

~~$-3.99\ cm$~~

$r = 3.99\ cm$