



Interest rates over different time periods and effective interest rates

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to convert nominal (annual) interest rates to compounding period interest rates.
- To be able to model loans with different compounding periods using recurrence relations.
- To be able to model investments with different compounding periods using recurrence relations.
- To be able to compare loans and investments using effective interest rates.
- To be able to calculate effective interest rates using a CAS calculator.



Recap

In previous lessons we have been looking at how we can create terms in a sequence using either a recurrence relation or a rule. We learned that it is possible to convert a recurrence relation into a rule and vice-versa. We learned about arithmetic sequences and geometric sequences and how we can create them.

Once the main theory was understood we then applied this to finance and learned about simple interest, compound interest, flat rate depreciation, unit cost depreciation, and reducing balance depreciation.

In each example the rate of interest was (generally) given as a per annum value (per year).

It is rare for a bank account to actually pay interest once per year.



Interest rate conversions

Interest rates are (generally) expressed as an annual rate. They are normally in nice and big print.

Note: It's the small print you want to worry about!!!

However, banks can actually pay interest quarterly, monthly, or daily. They don't advertise these rates as they believe you can calculate them yourself.



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Interest rate conversions

It's pretty simple to convert interest rates. All you do is take the annual interest rate and divide it by:

Months:	12
Fortnights:	26
Quarters:	4
Days:	365

For example, an annual interest rate of 12% would be a monthly interest rate of:

$$\frac{12\%}{12} = 1\%$$

This would be the value of r we would then use in any equations

Exam hints:

Interest rate conversions are used extensively in the General Maths exam and SACs.

It's important you know how to decode the question and convert annual interest rates into other time periods.



Example: Interest rate conversions

An investment account will pay interest at the rate of 4.68% per annum. Convert this interest rate to each of the following rates:

a monthly

b fortnightly

c quarterly.

$$\frac{4.68\%}{12} = \underline{\underline{0.39\%}}$$

$$\frac{4.68\%}{26} = 0.18\%$$

$$\frac{4.68\%}{4} = 1.17\%$$

Exam hints:

You can also do conversions the other way!
Given a quarterly rate, you can work out an annual rate.



Example: Recurrence relations with different compounding periods

Brian borrows \$5000 from a bank. He will pay interest at the rate of 4.5% per annum.

Let V_n be the value of the loan after n compounding periods.

Write down a recurrence relation to model the value of Brian's loan if interest is compounded:

- a** yearly **b** quarterly **c** monthly.

$$V_0 = 5000, \quad V_{n+1} = 1.045 V_n$$

Language hint:

A compounding period is another way of describing how often interest is being added or charged.

This will generally mean you will need to change a per annum interest rate into something else.

$$r = 4.5$$

$$R = 1 + \frac{4.5}{100}$$

$$1.045$$



Example: Recurrence relations with different compounding periods

Brian borrows \$5000 from a bank. He will pay interest at the rate of 4.5% per annum.

Let V_n be the value of the loan after n compounding periods.

Write down a recurrence relation to model the value of Brian's loan if interest is compounded:

a yearly

b quarterly

c monthly.

$$V_0 = 5000, \quad V_{n+1} = 1.01125 \times V_n$$

Language hint:

A compounding period is another way of describing how often interest is being added or charged.

This will generally mean you will need to change a per annum interest rate into something else.

$$r = \frac{4.5\%}{4}$$

$$r = 1.125$$

$$R = 1 + \frac{r}{100}$$

$$R = 1 + \frac{1.125}{100}$$



Example: Recurrence relations with different compounding periods

Brian borrows \$5000 from a bank. He will pay interest at the rate of 4.5% per annum.

Let V_n be the value of the loan after n compounding periods.

Write down a recurrence relation to model the value of Brian's loan if interest is compounded:

a yearly **b** quarterly **c** monthly.

$$V_0 = 5000, \quad V_{n+1} = 1.00375 \times V_n$$

Language hint:

A compounding period is another way of describing how often interest is being added or charged.

This will generally mean you will need to change a per annum interest rate into something else.

$$r = \frac{4.5\%}{12} \\ = 0.375$$

$$R = 1 + \frac{0.375}{100} \\ = 1.00375$$



Example: Different compounding periods, recurrence relations and rules

A principal value of \$10 000 is invested in an account earning **compound interest monthly** at the **rate of 9% per annum**.

Let V_n be the value of the investment after n months.

- a** Calculate the growth multiplier, R .
- b** Write down a recurrence relation for the value of the investment after n months.
- c** Write down a rule for the value of the investment after n months.
- d** Use this rule to find the value of the investment after 4 years.

$$a) \quad R = 1.0075$$

$$b) \quad V_0 = 10\,000, \quad V_{n+1} = 1.0075 \times V_n$$

$$c) \quad V_n = 10\,000 \times 1.0075^n$$

$$d) \quad V_{48} = 10\,000 \times 1.0075^{48} \\ = \$14\,314.05$$

$$r = \frac{9}{12}$$

$$R = 1 + \frac{0.75}{100}$$

$$= 1.0075$$



Calculating the rate of interest

Working out the percentage rate of interest is really important in General maths and is used extensively when we get to Amortisation tables.

If we want to know the rate of interest, you use the following formula:

$$\text{rate of interest} = \frac{\text{interest added}}{\text{initial amount before interest}} \times 100$$

It is really important to note that if the interest is added monthly, then the rate you calculate will be a monthly rate. If the interest payment is quarterly, the rate of interest you calculate will be quarterly. You would then need to convert this to an annual rate of interest.



Effective interest rates

Let's look at what happens when we invest money in four different accounts. One pays interest yearly, one pays interest quarterly, one monthly and the final one daily. No additional money is being added during the year.

We notice that the amounts at the end of the year are different.

This is because we are **compounding the interest**. We are being paid interest on the interest.

Opening amount	6000
Annual Rate of interest	9
Quarterly interest	2.25
Monthly interest	0.75

$$p - a = 9\%$$

	Annual	Quarterly	Monthly	Daily
Month	\$ 6,000.00	\$ 6,000.00	\$ 6,000.00	\$ 6,000.00
1			\$ 6,045.00	
2			\$ 6,090.34	
3		\$ 6,135.00	\$ 6,136.02	
4			\$ 6,182.04	
5			\$ 6,228.40	
6		\$ 6,273.04	\$ 6,275.11	
7			\$ 6,322.18	
8			\$ 6,369.59	
9		\$ 6,414.18	\$ 6,417.37	
10			\$ 6,465.50	
11			\$ 6,513.99	
12	\$ 6,540.00	\$ 6,558.50	\$ 6,562.84	\$ 6,564.97



Calculating: Effective interest rates

Aren't we being paid the same amount of interest though?

Yes. The **annual rate** is the same, but because we are being paid interest more regularly, the money grows quicker.

We can work out the **effective interest rate** which is the “actual” annual rate of interest we are really getting (or being charged).

$$\frac{562.84}{6000} \times 100$$

Opening amount	6000
Annual Rate of interest	9
Quarterly interest	2.25
Monthly interest	0.75

	Annual	Quarterly	Monthly	Daily
Month	\$ 6,000.00	\$ 6,000.00	\$ 6,000.00	\$ 6,000.00
1			\$ 6,045.00	
2			\$ 6,090.34	
3		\$ 6,135.00	\$ 6,136.02	
4			\$ 6,182.04	
5			\$ 6,228.40	
6		\$ 6,273.04	\$ 6,275.11	
7			\$ 6,322.18	
8			\$ 6,369.59	
9		\$ 6,414.18	\$ 6,417.37	
10			\$ 6,465.50	
11			\$ 6,513.99	
12	\$ 6,540.00	\$ 6,558.50	\$ 6,562.84	\$ 6,564.97
Effective	9.00	9.31	9.38	9.42



Calculating: Effective interest rates

Remember, the rate of interest is given by:

$$\text{rate of interest} = \frac{\text{interest added}}{\text{initial amount before interest}} \times 100$$

This requires us to know the end amount of the investment or loan.

Can we do it in another way?

12	\$ 6,540.00	\$ 6,558.50	\$ 6,562.84	\$ 6,564.97
Effective	9.00	9.31	9.38	9.42

$$\frac{558.50}{6000} \times 100$$



Calculating Effective interest rates: Using the CAS

Your CAS has a menu function which will allow effective rates of interest to be calculated given a **nominal rate of interest**.

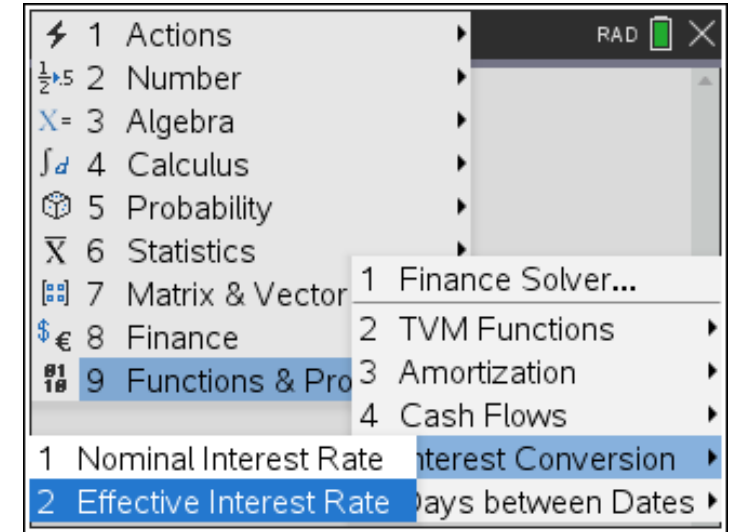
Note: A nominal rate of interest is the per annum rate.

$\text{eff}(\text{nominal rate of interest, compounding periods})$

$\text{convEff}(\text{compounding periods, nominal rate of interest})$

$$\text{eff}(9, 1)$$

$$\text{eff}(9, 4) = 9.31\%$$



Calculating effective interest rates: Using the CAS

Hence we can calculate the effective interest rates from our own example using the CAS function :

12	\$ 6,540.00	\$ 6,558.50	\$ 6,562.84	\$ 6,564.97
Effective	9.00	9.31	9.38	9.42

1.1	*Doc	RAD	✕
eff(9,1)		9.	
eff(9,4)		9.30833187891	
eff(9,12)		9.3806897671	
eff(9,365)		9.416214493	



Calculating nominal interest rates: Using the CAS

We can also do the reverse! Given the effective interest rate we can find the nominal (annual) interest rate.

12	\$ 6,540.00	\$ 6,558.50	\$ 6,562.84	\$ 6,564.97
Effective	9.00	9.31	9.38	9.42

1.1	*Doc	RAD	✕
nom(9,1)	9.		
nom(9.31,4)	9.00156039712		
nom(9.38,12)	8.99936465718		
nom(9.42,365)	9.00346052467		



Calculating nominal interest rates: Using a formula

There is a formula which we can use to find the effective interest rate given:

- r be the nominal interest rate per annum
- R_{eff} be the effective annual interest rate
- n be the number of times the interest compounds each year.

The formula is pretty disgusting to look at!

$$r_{eff} = \left(\left(1 + \frac{r}{100 \times n} \right)^n - 1 \right) \times 100\%$$



Example

Brooke would like to borrow \$20 000 that she will repay entirely after one year. She is deciding between two loan options:

- option A: 5.95% per annum, compounding weekly
- option B: 6% per annum, compounding quarterly.

$$6.13\%$$

$$6.14\%$$

- Calculate the effective interest rate for each investment.
- Which investment option is the best and why?



Example

Marissa has \$10 000 to invest. She chooses an account that will earn compounding interest at the rate of 4.5% per annum, compounding monthly.

Use a CAS calculator to find the effective interest rate for this investment, correct to three decimal places.

$$\text{eff}(4.5, 12) = \underline{\underline{4.594\%}}$$



Question 28

The nominal interest rate for a loan is 8% per annum.

When rounded to two decimal places, the effective interest rate for this loan is **not**

- ~~A.~~ 8.33% per annum when interest is charged daily.
- ~~B.~~ 8.32% per annum when interest is charged weekly.
- C.** 8.31% per annum when interest is charged fortnightly.
- D. 8.30% per annum when interest is charged monthly.
- E. 8.24% per annum when interest is charged quarterly.



VCAA Questions

VCAA 2020 Further Maths
Exam 2

Question 20

Consider the following amortisation table for a reducing balance loan.

Payment number	Payment	Interest	Principal reduction	Balance
0	0.00	0.00	0.00	300 000.00
1	1050.00	900.00	150.00	299 850.00
2	1050.00	899.55	150.45	299 699.55
3	1050.00	899.10	150.90	299 548.65

$$\frac{900}{300000} \times 100 = 0.3\%$$

$$\frac{3.6}{?} = 0.3$$

$$\frac{3.6}{0.3} = 12$$

The annual interest rate for this loan is 3.6%.

Interest is calculated immediately before each payment.

For this loan, the repayments are made

- A. weekly.
- B. fortnightly.
- ☒ C. monthly.
- D. quarterly.
- E. yearly.



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