



Using an explicit rule for geometric growth or decay

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to write explicit rules for geometric growth and decay.
- To be able to use an explicit rule to find the value of an investment after n years.
- To be able to calculate the value and total depreciation of an asset after a period of reducing balance depreciation.
- To be able to use a calculator to solve geometric growth and decay problems



Recap

In the previous lessons we have been looking at linear growth and geometric growth. These are really important and feature heavily in the exam. Understanding how to use them is critical to success.

In the last lesson we looked at how to model geometric growth and decay. We looked primarily at using recurrence relations. Like we did with linear growth, we can convert geometric recurrence relations into geometric rules.

This will allow us to stop going from term to term and allow us to go straight to a particular number in the sequence. This makes life a lot easier!



Turning a recurrence relation into a rule

In a previous lesson looking at Arithmetic growth and decay we converted a recurrence relation into a rule. We can use exactly the same process for a geometric sequence.

Let's turn the following recurrence relation into a rule:

$$V_0 = 200, \quad V_{n+1} = 2 * V_n$$

$$\begin{aligned} V_0 &= 200 & = 200 \times 2^0 \\ V_1 &= 200 \times 2 & = 200 \times 2^1 \\ V_2 &= 200 \times 2 \times 2 & = 200 \times 2^2 \\ V_3 &= 200 \times 2 \times 2 \times 2 & = 200 \times 2^3 \\ V_n &= & = V_0 \times 2^n \end{aligned}$$



A rule for individual terms of a geometric growth and decay sequence

This is going to be great for your summary book.

Given the following general form of a recurrence relation:

$$V_0 = P, \quad V_{n+1} = R \times V_n$$

We can use the following rule to get us any term in the sequence:

$$V_n = R^n \times V_0$$

Exam hints:

Students tend to lose lots of marks by not being able to understand the difference between a recurrence relation and a rule.



Example: Writing explicit rules

Write down **a rule** for the value of V_n in terms of n for each of the following. Use the rule to find the value of V_6 .

a $V_0 = 5, \quad V_{n+1} = 4V_n$

b $V_0 = 10, \quad V_{n+1} = 0.5V_n$

a)
$$V_n = 4^n \times 5$$
$$V_6 = 4^6 \times 5$$
$$= \underline{20\,480}$$

b)
$$V_n = 10 \times 0.5^n$$
$$V_6 = 10 \times 0.5^6$$
$$= \underline{0.15625}$$

Exam hints:

Students tend to lose lots of marks by not being able to understand the difference between a recurrence relation and a rule.



Compound interest loans and investments

When looking at how to apply the theory to real world situations, we found that, for compound interest loans and investments **interest is always added on**. We had the following as the recurrence relationships:

$$V_0 = P, \quad V_{n+1} = R \times V_n, \quad \text{where } R = 1 + \frac{r}{100}$$

We can now write this as a rule to allow us to find the value of any loan or investment at any time.

$$V_n = \left(1 + \frac{r}{100}\right)^n \times V_0$$

$$\begin{aligned} V_n &= R^n \times V_0 \\ &= \left(1 + \frac{r}{100}\right)^n \times V_0 \\ &= \end{aligned}$$



Example: Compound interest loans and investments

The rule for the value of the investment after n years, V_n , is shown below.

$$V_n = 1.09^n \times 10\,000$$

- a State how much money was initially invested. $\$10\,000$
- b Find the annual interest rate for this investment. 9%
- c Find the value of the investment after 4 years, **correct to the nearest cent.**
- d Find the amount of interest earned over the first 4 years, correct to the nearest cent.
- e Find the amount of interest earned in the fourth year, correct to the nearest cent.
- f Determine if the investor has doubled their money within 10 years.

$$\begin{aligned} V_4 &= 1.09^4 \times 10\,000 \\ &= \$14\,115.82 \end{aligned}$$

$$R = 1 + \frac{r}{100}$$

$$\therefore 1.09 = \cancel{1} + \frac{r}{100} \quad -1$$

$$\times 100 \quad 0.09 = \frac{r}{100} \quad \times 100$$

$$r = \underline{\underline{9\%}}$$



Example: Compound interest loans and investments

The rule for the value of the investment after n years, V_n , is shown below.

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- a** State how much money was initially invested.
- b** Find the annual interest rate for this investment.
- c** Find the value of the investment after 4 years, correct to the nearest cent.
- d** Find the amount of interest earned over the first 4 years, correct to the nearest cent.
- e** Find the amount of interest earned in the fourth year, correct to the nearest cent.
- f** Determine if the investor has doubled their money within 10 years.

$$4^{\text{th}} : 14\,115.82$$

$$3^{\text{rd}} : 12\,950.29$$

$$V_{10} = 23\,673.64$$

$$\begin{aligned} & 14\,115.82 - 10\,000 \\ &= \underline{\underline{\$4\,115.82}} \\ \therefore \text{Int} &= \underline{\underline{\$4\,115.82}} \end{aligned}$$



Reducing balance depreciation

When we are subject to reducing balance depreciation we need to remember that the value of the purchase **will always go down**. We had the following as the recurrence relationships:

$$V_0 = P, \quad V_{n+1} = R \times V_n, \quad \text{where } R = 1 - \frac{r}{100}$$

We can now write this as a rule to allow us to find the value of any loan or investment at any time. **Notice the negative sign.**

$$V_n = \left(1 - \frac{r}{100}\right)^n \times V_0$$

Exam hints:

Students tend to have formula overload and can, at times, use the wrong one.

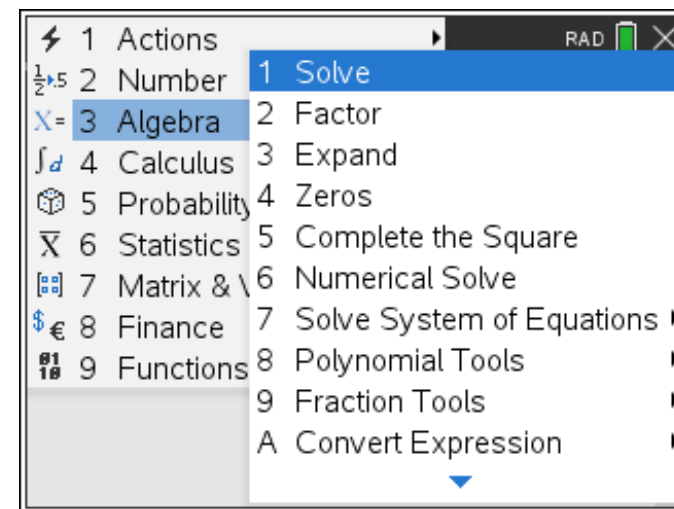


Using a CAS for more complex problems

The CAS is great for more complex problems.

In the previous examples we have been only been interested we have always been given the time period in which to find values of loans and investments. What is we were given the start and end values of a loan or investment and had to find how long it took to get there?!

Note: You need to know how to use the SOLVE function of the CAS.



Example: Using a CAS to find the value of n

How many years will it take for an investment of \$5000, paying compound interest at 6% per annum, to grow above \$8000? Write your answer correct to the nearest year.

Exam hints:

Sadly, a lot of students do poorly in General Maths as they find it hard to comprehend what is being asked in the questions. General Maths is very, very heavy in language – particularly in the module on Finance and Recursion.

$$n? \quad V_0 = 5000 \quad r = 6\%$$

$$V_n = 8000$$

$$V_n = V_0 \times R^n$$

$$8000 = 5000 \times 1.06^n$$

$$n = 8.066 \dots$$

$$n = \underline{\underline{9}}$$

$$R = 1 + \frac{6}{100}$$

$$= 1 + 0.06$$

$$= 1.06$$

$$V_n = 5000 \times 1.06^n$$



Example: Using a CAS to find the value of r

An industrial weaving company purchased a new loom at a cost of \$56 000. It has an estimated value of \$15 000 after 10 years of operation. If the value of the loom is depreciated using a reducing balance method, what is the annual rate of depreciation?

Write your answer correct to one decimal place.

$$V_0 = 56\,000$$

$$V_n = 15\,000$$

$$n = 10$$

$$V_n = V_0 \times R^n$$

$$15000 = 56000 \times R^{10}$$

$$R = 0.876577..$$

$$r = \underline{12.3}\%$$

Exam hints:

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$$R = 1 - \frac{r}{100}$$

$$0.876577 = 1 - \frac{r}{100}$$



Question 22

Tim deposited \$6000 into an investment account earning compound interest calculated monthly.

A rule for the balance, T_n , in dollars, after n years is given by $T_n = 6000 \times 1.003^{12n}$.

Let R_n be a new recurrence relation that models the balance of Tim's account after n months.

This recurrence relation is

~~A.~~ $R_0 = 6000, R_{n+1} = R_n + 18$

~~B.~~ $R_0 = 6000, R_{n+1} = R_n + 36$

C. $R_0 = 6000, R_{n+1} = 1.003R_n$

D. $R_0 = 6000, R_{n+1} = 1.0036R_n$

E. $R_0 = 6000, R_{n+1} = 1.036R_n$

$$T_n = 6000 \times 1.003^{12n}$$

$$R_0 = 6000, R_{n+1} = 1.003 \times R_n$$



VCAA Questions

VCAA 2021 Further Maths
Exam 1

Question 20

Sammy purchased a boat for \$72 000.

The value of the boat is depreciated each year by 10% using the reducing balance method.

In the third year, the boat will depreciate in value by 10% of

- A. \$47 239.20
- B. \$52 488.00
- ☒ C. \$58 320.00
- D. \$64 800.00
- E. \$72 000.00

$$\begin{aligned}V_n &= V_0 \times R^n \\V_2 &= 72\,000 \times 0.9^2 \\&= \underline{\underline{\$58\,320}}\end{aligned}$$

Value 2 years

$$\begin{aligned}R &= 1 - \frac{10}{100} \\&= 1 - 0.1 \\&= \underline{\underline{0.9}}\end{aligned}$$



VCAA Questions

Question 26

Ray deposited \$5000 in an investment account earning interest at the rate of 3% per annum, compounding quarterly.

A rule for the balance, R_n , in dollars, after n years is given by

- A. $R_n = 5000 \times 0.03^n$
- B. $R_n = 5000 \times 1.03^n$
- C. $R_n = 5000 \times 0.03^{4n}$
- D. $R_n = 5000 \times 1.0075^n$
- E. $R_n = 5000 \times 1.0075^{4n}$**

$$R_n = R_0 \times R^n$$

$$R_n = 5000 \times 1.0075^{4n}$$

$$r = Q = \frac{3}{4} = \underline{\underline{0.75\%}}$$

$$R = 1 + \frac{0.75}{100}$$

$$= 1.0075$$



VCAA 2020 Further Maths
Exam 1

Note:

Questions don't have to just be written to tick one content area. They can span more than one content area.

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