



Transformations of graphs of functions

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Be able to find equations of images when given the transformations using substitution algebra.



RECAP

This is the final lesson in this series, and it brings all the learning together in one place. Having looked at the ideas of dilations, reflections and translations and how they can be used to move and stretch graphs, we then looked at how we can reverse the process and find the transformations which had been applied to a base graph.

We now look at how we can present the information relating to transformations in different ways (to try and trick you!).



Applying transformation rules to graphs

I hate to recreate good material when it already exists out there, and here is a very good summary of the work we are going to need to use in this section of the course

Mapping	Rule	The graph of $y = f(x)$ maps to
Reflection in the x -axis	$(x, y) \rightarrow (x, -y)$	$y = -f(x)$
Reflection in the y -axis	$(x, y) \rightarrow (-x, y)$	$y = f(-x)$
Dilation of factor a from the y -axis	$(x, y) \rightarrow (ax, y)$	$y = f\left(\frac{x}{a}\right)$
Dilation of factor b from the x -axis	$(x, y) \rightarrow (x, by)$	$y = bf(x)$
Translation of h units in the positive direction of the x -axis and k units in the positive direction of the y -axis	$(x, y) \rightarrow (x + h, y + k)$	$y - k = f(x - h)$

It's important to note that the Methods course is all about understanding the Mathematical language being used. If you understand the language, then the mathematics becomes much easier!



Example 1

A transformation is defined by the rule $(x, y) \rightarrow \left(\frac{1}{3}x, 2y\right)$.

Find the equation of the image of the graph of $y = x^2 + 2x + 3$ under this transformation.

image

$$\frac{y}{2} = \frac{(3x)^2 + 2(3x) + 3}{2}$$
$$\frac{y}{2} = 9x^2 + 6x + 3$$
$$y = \underline{\underline{18x^2 + 12x + 6}}$$

$$\left(\frac{1}{3}x, 2y\right)$$

x' y'

$$\therefore \frac{1}{3}x = x' \quad 2y = y'$$
$$\underline{\underline{x = 3x'}} \quad \underline{\underline{y = \frac{y'}{2}}}$$



Example 2

A transformation is defined by the rule

$$(x, y) \rightarrow (-3y + 1, 2x + 2)$$

Find the image of the straight line with equation line $y = 2x + 5$ under this transformation.

$$\frac{x' - 1}{-3} = \cancel{2} \left(\frac{y' - 2}{\cancel{2}} \right) + 5$$

$$\frac{x' - 1}{-3} = y' - 2 + 5$$

$$x' - 1 = -3y' + 6 - 15 + 1$$

$$x' = -3y' - 8$$

$$-3y + 1 \quad 2x + 2$$

$$x' \quad y'$$

$$-3y + 1 = x' \quad 2x + 2 = y'$$

$$-3y = x' - 1 \quad 2x = y' - 2$$

$$y = \frac{x' - 1}{-3} \quad x = \frac{y' - 2}{2}$$

$$x + 8 = -3y$$

$$y = \frac{x + 8}{-3}$$



Example 3

Find a sequence of transformations that takes the graph of the quadratic function with rule $f_1(x) = x^2 + 2x + 6$ to the graph of the quadratic function with rule $f_2(x) = 2x^2 + 8x + 5$.

$$x \rightarrow x + 1$$

$$4 \rightarrow 2.5$$

$$-1.5$$

D.

R,

T.

$$y = (x+1)^2 + 5 \quad f_2(x) = 2(x^2 + 4x + 5/2)$$

$$= 2[(x+2)^2 - 3/2]$$

$$\frac{y}{2} = (x+1)^2 + 5 \rightarrow = 2(x+2)^2 - 3$$

$$\frac{y}{2} = (x+1)^2 + 5$$

$$y = 2(x+1)^2 + 10$$

$$y + 13 = 2(x+1)^2 + 10 - 3$$

$$y = 2(x+2)^2 - 3$$

1. • Dil factor 2 from x axis

1. $\therefore$ Trans 13 ↓

1. • Trans! 1 ←



Example 4

Let $f(x) = x^2$.

- a** Write the equation for the graph of $y = 4f(x + 3)$.
b Give a sequence of transformations that takes the graph of $y = f(x)$ to the graph of $y = 4f(x + 3)$.

a. $f(x) = x^2$
 $4f(x) = 4x^2$
 $4f(x+3) = 4\underline{(x+3)}^2$

b. $x \rightarrow x + 3$

$$y = 4f(x+3)$$

$$\underline{\underline{y = x^2}}$$

$$\frac{y}{4} = x^2$$

$$y = 4x^2$$

$\therefore$ Dilation factor
4 from x -axis

$\therefore$ Translation
3 $\leftarrow$



Example 5a

The points (1, 1) and (3, 9) lie on the graph of $y = x^2$. A transformation with a rule of the form $(x, y) \rightarrow (ax + h, by + k)$ maps $(1, 1) \rightarrow (5, 4)$ and $(3, 9) \rightarrow (11, 12)$.

a Find the values of a , h , b and k for this transformation.

b Find the image of $y = x^2$ under this transformation.

$$(1, 1) \rightarrow (5, 4)$$

$$x \quad y \quad x' \quad y'$$

$$(3, 9) \rightarrow (11, 12)$$

$$a = 3$$

$$b = 1$$

$$h = 2$$

$$k = \underline{\underline{3}}$$

$$x' = ax + h$$

$$5 = a \cdot 1 + h$$

$$a + h = 5$$

$$b + k = 4$$

$$11 = a \cdot 3 + h$$

$$3a + h = 11$$

$$\begin{array}{r} 3a + h = 11 \\ - \quad a + h = 5 \\ \hline 2a = 6 \end{array}$$

$$2a = 6$$

$$a = \underline{\underline{3}}$$

$$(ax + h, by + k)$$
$$x' \quad y'$$

$$y' = by + k$$

$$4 = b \cdot 1 + k$$

$$12 = b \cdot 9 + k$$

$$9b + k = 12$$

$$- \quad b + k = 4$$

$$\hline 8b = 8$$

$$\underline{\underline{b = 1}}$$



Example 5b

The points (1, 1) and (3, 9) lie on the graph of $y = x^2$. A transformation with a rule of the form $(x, y) \rightarrow (ax + h, by + k)$ maps $(1, 1) \rightarrow (5, 4)$ and $(3, 9) \rightarrow (11, 12)$.

a Find the values of a , h , b and k for this transformation.

b Find the image of $y = x^2$ under this transformation.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a = 3 \quad h = 2$$

$$b = 1 \quad k = 3$$

$$y = x^2$$

$$y' - 3 = \left(\frac{x' - 2}{3}\right)^2$$

$$y = \frac{(x - 2)^2}{9} + 3$$

$$(x, y)$$

$$\rightarrow (3x + 2, y + 3)$$

$$x' \quad y'$$

$$3x + 2 = x' \quad y + 3 = y'$$

$$3x = x' - 2 \quad y = \underline{\underline{y' - 3}}$$

$$x = \frac{x' - 2}{3}$$



Learning Objectives: Reviewed

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