

Modelling geometric growth and decay

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to graph the terms of a geometric sequence.
- To be able to model compound interest with a recurrence relation.
- To be able to model reducing balance depreciation with recurrence relations.
- To be able to use reducing balance depreciation with recurrence relations.



Recap

In the previous lessons we have been looking at linear growth and decay. This is where the value of an investment or loan or the depreciation of something goes up or down in fixed amounts. This is all well and good in an ideal world, but isn't how the real world works.

When I pay money into my account I don't want a fixed amount of interest. I want interest to be paid on the interest please.

When this happens, it means the money grows (and drops) geometrically. In the following formula, which we have been using and adapting all though the series, the value of R is no longer going to be 1 (one).

$$V_0 = P, \qquad V_{n+1} = R \times V_n \pm D$$

Start term

Next term

Geometric growth or decay multiplier

Current term

Fixed amount added or subtracted

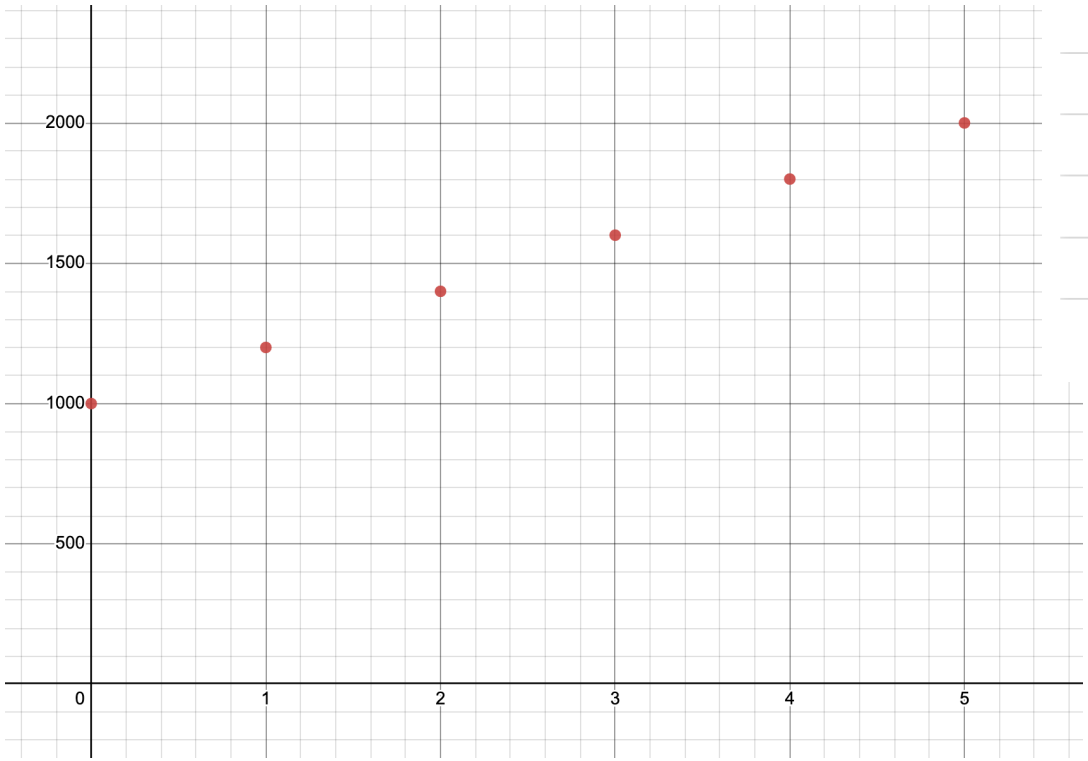


Showing geometric growth

When we have the following relation, we saw that the value of the investment went up $200 per period. Or, the sequence went up by 200 each time.

$$V_0 = 1000, \qquad V_{n+1} = V_n + 200$$

The value of 'R' was one. This meant the graph was linear.



n	v
0	1000
1	1200
2	1400
3	1600
4	1800
5	2000



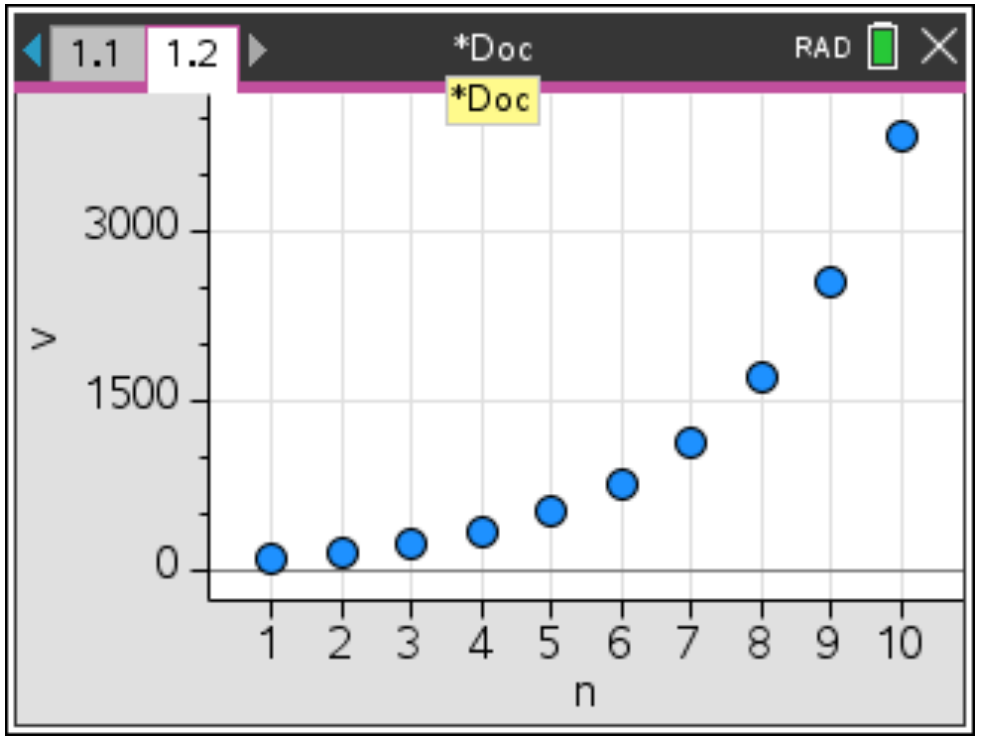
Showing geometric growth

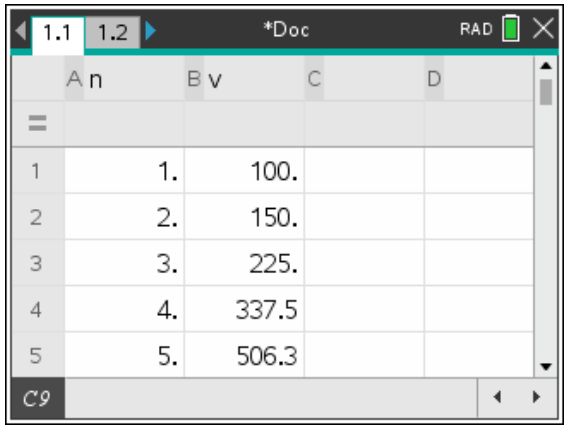
When we don't have a value for 'D" and we have an R value which is greater than 1 (one) we see that the dots now curve up!

$$V_0 = 100, \qquad V_{n+1} = 1.5V_n$$

Really important:

Geometric growth is only possible when the value of D is zero.
Things will still grow, but it cannot be described as geometric.







Showing geometric growth

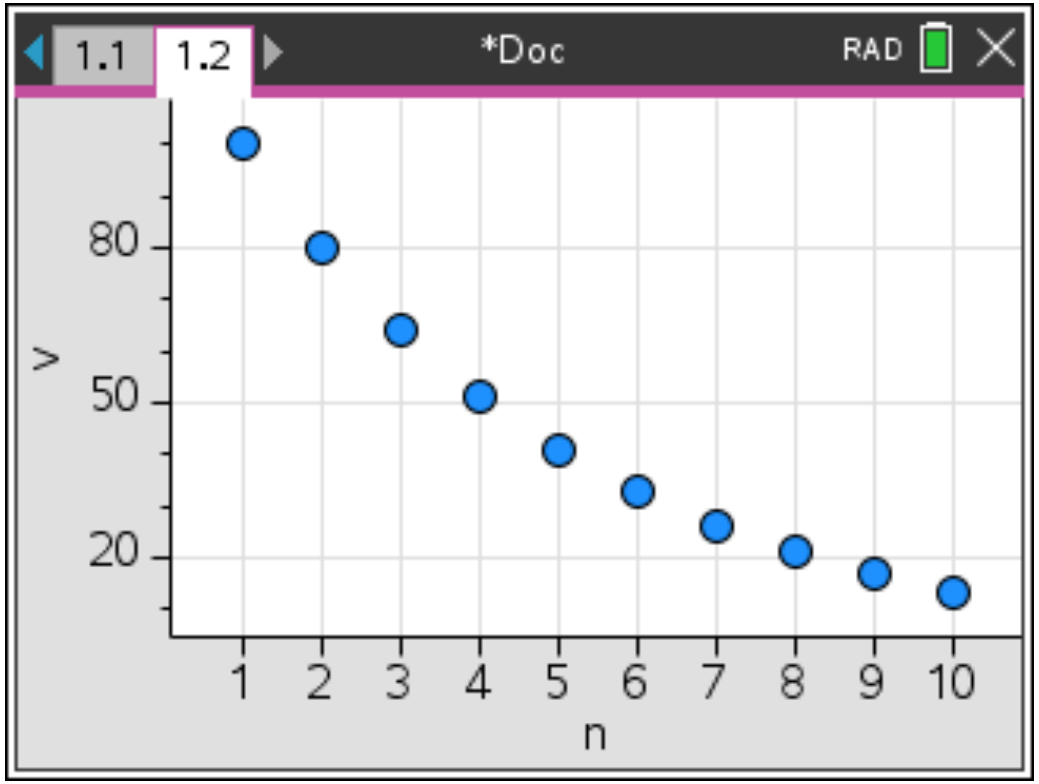
When the value of R is between 0 and 1, then dots will curve down.

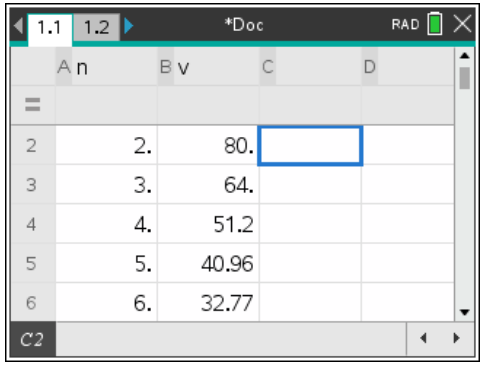
$V_0 = 100, \qquad V_{n+1} = 0.8V_n$

1 − 0·8 = 0·2

× 100

20%





Really important:

Geometric growth is only possible when the value of D is zero.
Things will still grow, but it cannot be described as geometric.



Real world examples: Compound interest and loans

In the real world there is no such thing (really) as simple interest. When we pay money into the bank we expect to get interest.

If I have $1000 and pay it into an account earning 10% interest per month, I want will have:

10% of 1000 = $100 in interest

So, at the end of the first month I would have $1100.

I now want interest on the $1100 which will be:

10% of 1100 = $110 in interest

So, in the second month I have $10 more in interest.

$1100 10%

10% of $1100 = 110

1100
110
$1210

Note:

When interest is paid on the interest, we say that it is **compounding**.

Interest compounds on both investments and loans.



From percentages to decimal equivalence

The value of R is the following equation is always a decimal value.

$$V_0 = principal, \qquad V_{n+1} = RV_n$$

But interest is generally given as a percentage. So, we need to be able to convert from a percentage into a R value.

For example, a 10% increase will have an R of 1.1

A 20% decrease will have an R of 0.8.

10% ↑ 100%

110% of ...

$\frac{110}{100} = 1.1$

30% ↓

100% − 30%

= 70%

$\frac{70}{100} = 0.7$



Compound interest investments and loans

For those who love a formula, we can use the following to help us:

$$V_0 = initial\ value, \qquad V_{n+1} = RV_n, \qquad where\ R = 1 + \frac{r}{100}$$

This is the **annual** percentage interest rate

We are adding this on as interest is being added on when we save or take out a loan.



Example

The following recurrence relation can be used to model a compound interest investment of $2000 paying interest at the rate of 7.5% per annum.

$$V_0 = 2000, \qquad V_{n+1} = 1.075V_n$$

In the recurrence relation, V_n is the value of the investment after n years.

- Use the recurrence relation to show that the value of the investment after 3 years is $2484.59.
- Determine when the value of the investment will first exceed $2500.

$100 + 7.5\%$

107.5%

100

1.075

$V_0 = 2000$

$V_1 = 1.075 \times 2000 = 2150$

$V_2 = 1.075 \times 2150 = 2311.25$

$V_3 = 1.075 \times 2311.25 = \$2484.59$

4 years

$V_0 = initial\ value, \qquad V_{n+1} = RV_n, \qquad where\ R = 1 + \frac{r}{100}$



Reducing balance depreciation

Although we seem to have lots and lots of names for things here in Mathematics, this is essentially the same as we have been doing except that when something depreciates, it does DOWN in value. This means the value of R will be between 0 and 1 (not inclusive).

So, we can use the following formula:

$$V_0 = initial\ value, \qquad V_{n+1} = RV_n, \qquad where\ R = 1 - \frac{r}{100}$$

Rate of depreciation per annum

Negative because the value is reducing



Example

A sofa was purchased for $7500 and is depreciating at a reducing balance rate of 8.4% per annum. Write down a recurrence relation where Vn is the value of the sofa after n years.

$$V_0 = 7500, \quad V_{n+1} = 0.916 \cdot V_n$$

$$R = 1 - \frac{r}{100}$$

$$= 1 - \frac{8.4}{100}$$

$$= 0.916$$



Example

The following recurrence relation can be used to model the value of office furniture with a purchase price of $9600, depreciating at a reducing-balance rate of 7% per annum.

$$V_0 = 9600, \qquad V_{n+1} = 0.93V_n$$

In the recurrence relation, V_n is the value of the office furniture after n years.

- Use the recurrence relation to find the value of the office furniture, correct to the nearest cent, after 1, 2 and 3 years.
- If the office furniture was initially purchased in 2023, at the end of which year will the value of the investment first be less than $7000?

$R = 1 - \frac{7}{100}$

$= 0.93$

23

$V_0 = 9600$

$V_1 = 8928$

$V_2 = 8303.04$

$V_3 = 7721.83$

2028



VCAA Questions

VCAA 2020 Further Maths
Exam 1

Question 24

Manu invests $3000 in an account that pays interest compounding monthly.

The balance of his investment after n months, B_n, can be determined using the recurrence relation

$$B_0 = 3000, \quad B_{n+1} = 1.0048 \times B_n$$

The total interest earned by Manu's investment after the first five months is closest to

A. $57.60
B. $58.02
C. $72.00
D. $72.69
E. $87.44

3000
3072.69



VCAA 2020 Further Maths
Exam 2

Question 9 (3 marks)

Samuel opens a savings account.

Let B_n be the balance of this savings account, in dollars, n months after it was opened.

The month-to-month value of B_n can be determined using the recurrence relation shown below.

$$B_0 = 5000, \qquad B_{n+1} = 1.003B_n$$

a. Write down the value of B_4, the balance of the savings account after four months.
Round your answer to the nearest cent. 1 mark

$5060.27



VCAA 2020 Further Maths
Exam 2

Question 9 (3 marks)

Samuel opens a savings account.

Let B_n be the balance of this savings account, in dollars, n months after it was opened.

The month-to-month value of B_n can be determined using the recurrence relation shown below.

$$B_0 = 5000, \qquad B_{n+1} = 1.003B_n$$

b. Calculate the monthly interest rate percentage for Samuel's savings account. 1 mark

$R = 1 + \frac{r}{100}$

$1.003 = 1 + \frac{r}{100}$

$r = 0.3\%$





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