Determining transformations

Year 11
Mathematical Methods

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Learning Objectives

By the end of the lesson, I hope that you understand and can apply the following to a range of questions from the Year 11 Mathematical Methods course.

- Be able to use the theory from the previous lessons to find the transformations which have been applied to go from a “base graph” to its image (as presented in the questions).

RECAP

We spend a lot of time in lessons showing you how to do things in a very linear fashion. I call this the "going forwards" method. We tend to hope that having understood how to do it forwards, you would be able to then do it backwards.

But the brain isn't always wired this way! You needed to be taught to count backwards. It wasn't automatic.

So, let's look at how to use the theory on the right to reverse the process.

Note: This can also be done with substitution algebra!

For the graph of $y = f(x)$, we have the following four pairs of equivalent processes:

1
- Applying the **dilation from the x-axis** $(x, y) \to (x, by)$ to the graph of $y = f(x)$.
- Replacing y with $\frac{y}{b}$ in the equation to obtain $y = bf(x)$ and graphing the result.

2
- Applying the **dilation from the y-axis** $(x, y) \to (ax, y)$ to the graph of $y = f(x)$.
- Replacing x with $\frac{x}{a}$ in the equation to obtain $y = f\left(\frac{x}{a}\right)$ and graphing the result.

3
- Applying the **reflection in the x-axis** $(x, y) \to (x, -y)$ to the graph of $y = f(x)$.
- Replacing y with $-y$ in the equation to obtain $y = -f(x)$ and graphing the result.

4
- Applying the **reflection in the y-axis** $(x, y) \to (-x, y)$ to the graph of $y = f(x)$.
- Replacing x with $-x$ in the equation to obtain $y = f(-x)$ and graphing the result.

Example 1a

a Find a sequence of transformations which takes the graph of $y = x^2$ to the graph of $y = 2(x-2)^2 + 3$.

b Find a sequence of transformations which takes the graph of $y = \sqrt{x}$ to the graph of $y = \sqrt{5x-2}$.

The best way to teach this is by example. So here are a couple of examples.

Remember, when they don't ask for a order you must use DrT

$y = x^2$

$\frac{y}{2} = x^2$

$y = 2x^2$

$y = 2(x-2)^2$

$y - 3 = 2(x-2)^2$

$y = 2(x-2)^2 + 3$

1. Dilation factor 2 from x-axis

1. Translation 2 units right

1. Translated 3 units up.

Example 1b

a Find a sequence of transformations which takes the graph of $y = x^2$ to the graph of $y = 2(x-2)^2 + 3$.

b Find a sequence of transformations which takes the graph of $y = \sqrt{x}$ to the graph of $y = \sqrt{5x-2}$.

$y = \sqrt{x}$

$y = \sqrt{\frac{x}{1/5}} = \sqrt{5x}$

$y = \sqrt{5(x-2/5)}$

$y = \sqrt{5x-2}$

The best way to teach this is by example. So here are a couple of examples.

Remember, when they don't ask for a order you must use DrT

D.R.T.

$x \to \frac{x}{1/5} \to 5x$

∴ Dilation factor 1/5 from y-axis 1.

∴ Translation 2/5 units to the right 1.

Example 2a

a Find a sequence of transformations which takes the graph of $y = \frac{3}{(x-1)^2} + 6$ to the graph of $y = \frac{1}{x^2}$.

b Find a sequence of transformations which takes the graph of $y = (5x-1)^2 + 6$ to the graph of $y = x^2$.

The best way to teach this is by example. So here are a couple of examples.

Remember, when they don't ask for a order you must use DrT

$y = \frac{3}{(x-1)^2} + 6$

$y \to y + 6$

$\therefore$ Translation of 6 units $\downarrow$

$y + 6 = \frac{3}{(x-1)^2} + 6$

$y \to \frac{y}{1/3} \to 3y$

$\therefore$ Dilation factor $1/3$ from x-axis

$y = \frac{3}{(x-1)^2}$

$y = \frac{1}{(x-1)^2}$

$x \to x + 1$

$\therefore$ Translation 1 unit $\leftarrow$

$3y = \frac{3 \cdot 1}{(x-1)^2}$

$y = \frac{1}{(x+1-1)^2} = \frac{1}{x^2}$

Example 2a

a Find a sequence of transformations which takes the graph of $y = \frac{3}{(x-1)^2} + 6$ to the graph of $y = \frac{1}{x^2}$.

b Find a sequence of transformations which takes the graph of $y = (5x-1)^2 + 6$ to the graph of $y = x^2$.

$y = \frac{3}{(x-1)^2} + 6$

$3y = \frac{3}{(x-1)^2} + 6 \times 2$

$y = \frac{1}{(x-1)^2} + 2$

$y + 2 = \frac{1}{(x-1)^2} + 2$

$y = \frac{1}{(x-1)^2}$

$y = \frac{1}{x^2}$

The best way to teach this is by example. So here are a couple of examples.

Remember, when they don't ask for a order you must use DrT

D.R.T.

$y \to \frac{y}{1/3} = 3y$

∴ Dilation 1/3 from x-axis

$y \to y + 2$

∴ Translation 2 ↑

$x \to x + 1$

∴ Trans 1 unit ←

Example 2b

a Find a sequence of transformations which takes the graph of $y = \frac{3}{(x-1)^2} + 6$ to the graph of $y = \frac{1}{x^2}$.

b Find a sequence of transformations which takes the graph of $y = (5x-1)^2 + 6$ to the graph of $y = x^2$.

$y = (5x-1)^2 + 6$

$y = \left(\frac{5 \cdot x}{5} - 1\right)^2 + 6$

$y = (x-1)^2 + 6$

$y + 6 = (x-1)^2 + 6$

$y = (x-1)^2$

$y = x^2$

$x \to \frac{x}{5}$

$\therefore$ Dilation factor 5 from y axis.

$\therefore$ Translated 6 units ↓

$x \to x+1$

$\therefore$ Tran 1 unit ←

The best way to teach this is by example. So here are a couple of examples.

Remember, when they don't ask for a order you must use DrT

Example 2b

a Find a sequence of transformations which takes the graph of $y = \frac{3}{(x-1)^2} + 6$ to the graph of $y = \frac{1}{x^2}$.

b Find a sequence of transformations which takes the graph of $y = (5x-1)^2 + 6$ to the graph of $y = x^2$.

$y = (5x-1)^2 + 6$

$y \neq 6 = (5x-1)^2 \neq 6$

$y = (5x-1)^2$

$y = [5(x - 1/5)]^2$

$y = (5x)^2$

$y = \left(\frac{x}{5}\right)^2$

$y = x^2$

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Remember, when they don't ask for a order you must use DrT

Trans 6 ↑

$x \to x + 1/5$

Tran 1/5 ←

$x \to \frac{x}{5}$

∴ Dilation factor 5 from y axis

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