Using an explicit rule for linear growth or decay

Year 12 General Maths
Units 3 and 4

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Learning Objectives

By the end of the lesson, I would hope that you have an understanding and be able to apply to questions the following concepts:

- To be able to convert a recurrence relation to an explicit rule.
- To be able to model a simple interest investment using an explicit rule.
- To be able to use a rule to determine the value of a simple interest loan or investment.
- To be able to model flat rate depreciation of an asset using an explicit rule.
- To be able to use a rule for the flat rate depreciation of an asset.
- To be able to use an explicit rule for unit cost depreciation.

Recap

In the previous lessons we have been looking at recurrence relations. These are used to create sequences (which can be used to model real world situations)

The problem with them is, we need to know the previous number and the relation, before we can get to the next number. I might be given a start number and the rule, but what if I wanted to know the value of V_{100} for example?

That would be tedious! Well, it turns out we can write recurrence relations as rules.

$$V_0 = P, \qquad V_{n+1} = R \times V_n \pm D$$

Fixed amount added or subtracted

Start term

Next term

Current term

Geometric growth or decay multiplier

Going from a relation to a rule

It is critically important that you understand that there is a big difference between a **recurrence relation** and a **rule**.

A recurrence relation will always start with V_0 and always have a relation.

$$V_0 = 1000, \qquad V_{n+1} = V_n + 200$$

A rule always starts with V_n.

$$V_n = V_0 + 200n$$

But if they give me one of them, how do I go to the other?

Common VCE mistakes:

Not knowing the difference between a relation and a rule.
Using the wrong one in questions and getting the wrong answer.

An worked example

If I have the following recurrence relation I can work out how to get to the rule by breaking the problem down into stages:

$$V_0 = 2000, \qquad V_{n+1} = V_n + 100$$

$V_0 = 2000 \qquad = V_0 + 0 \times 100$

$V_1 = 2000 + 100 \qquad = V_0 + 1 \times 100$

$V_2 = 2000 + 100 + 100 \qquad = V_0 + 2 \times 100$

$V_3 = 2000 + 100 + 100 + 100 \qquad = V_0 + 3 \times 100$

$V_n = V_0 + n.D$

Exam hint:

Make sure you show all working out when being asked to show the value at the end of a certain year.
Questions may only be worth one mark, but they do require some working out to be shown.

2000

2100

2200

2300

Going from the relation to the rule

So, we can see it’s possible, when being given the relation to go to the rule.

Example:

$$V_0 = 1000, \qquad V_{n+1} = V_n + 100$$

Can be changed into:

$$V_n = 1000 + 100n$$

Making it more general:

$$V_0 = initial\ value, \qquad V_{n+1} = V_n \pm D$$

This sign shows the equations for both growth and decay

$$V_n = V_0 + n \times D$$

Example: Converting a recurrence relation into a rule

Write down a rule for V_n for each of the following recurrence relations. Calculate V10 for each case.

- $V_0 = 8, \; V_{n+1} = V_n + 3$
- $V_0 = 400, \; V_{n+1} = V_n - 12$
- $V_0 = 30, \; V_{n+1} = V_n - 7$

$V_n = V_0 \pm n.D$

$V_0 \pm D.n$

$V_n = 8 + 3n$

$V_n = 400 - 12.n$

$V_n = 30 - 7.n$

Back to real world: Simple interest loans and investments

We have met the idea of simple interest before.

When we take out a loan, we have to pay the bank back money at a certain rate of interest. This interest rate is, once again, given as a percentage and is set as a fixed percentage of the amount you were loaned.

An investment is another way of saying that we are saving money.

We are going to use the following:

$$V_n = V_0 + nD, \qquad where\ D = \frac{r}{100} \times V_0$$

Example

Amie invests \$3000 in a simple interest investment with interest paid at the rate of 6.5% per year.

Use a rule to find the value of the investment after 10 years.

$$D = \frac{6.5}{100} \times 3000$$

$$= \$195$$

$$V_0 = 3000$$

$$V_n = 3000 + 195 \cdot n$$

$$V_{10} = 3000 + 195 \cdot 10$$

$$= \$4950$$

Example

The following recurrence relation can be used to model a simple interest investment:

$$V_0 = 3000, \qquad V_{n+1} = V_n + 260$$

where V_n is the value of the investment after n years.

- What is the principal of the investment?
- How much interest is added each year?
- Write down the rule for the value of the investment after n years.
- Use a rule to find the value of the investment after 15 years.
- Use a rule to find when the value of the investment first exceeds $10 000.

P = \$3000

int = \$260

$V_n = 3000 + 260n$

$V_{15} = 3000 + 260 \cdot 15$

$= \$6900$

$10\,000 = 3000 + 260 \cdot n$

$n = 26.923...$

n = <u>27</u> years

Back to flat rate depreciation

Here we are again. Maths is really repetitive.

Flat rate depreciation: The fixed amount by which something will reduce in value. This fixed rate will be set as a percentage of the starting value.

Example:
A photocopier costs $6000 when new. Its value depreciates at the flat rate of 17.5% per year. Write a **rule** and use this to find its value after 4 years.

$$V_n = V_0 - D.n$$

$$V_n = 6000 - 1050.n$$

$$V_4 = 6000 - 1050.4$$
$$= 1800$$

$$D = \frac{r}{100} \times V_0 = \frac{17.5}{100} \times 6000$$
$$= \$1050$$

Example

The following recurrence relation can be used to model the flat rate of depreciation of a set of office furniture:

$$V_0 = 12\,000, \qquad V_{n+1} = V_n - 1200$$

where V_n is the value of the furniture after n years.

- What is the initial value of the furniture?
- How much does the furniture decrease by each year?
- Write down the rule for the value of the investment after n years.
- Use a rule to find the value of the investment after 6 years.
- How long does it take for the furniture's value to decrease to zero?

$0 = 12000 - 1200 \cdot n$

$n = \underline{\underline{10}}$

\$12000

\$1200

$V_n = 12000 - 1200 \cdot n$

$V_6 = 12000 - 1200 \cdot 6$

$= \$4800$

Unit Cost Depreciation

This is beginning to feel a little repetitive isn't it! The is the same as flat rate deprecation (mathematically) it's just that the item reduces in value per use.

Example:
A hairdryer in a salon was purchased for $850. The value of the hairdryer depreciates by 25 cents for every hour it is in use.

Let V_n be the value of the hairdryer after n hours of use.

- Write down a rule to find the value of the hairdryer after n hours of use.
- What is the value of the hairdryer after 50 hours of use?
- On average, the salon will use the hairdryer for 17 hours each week. How many weeks will it take for the value of the hairdryer to halve?
- The hairdryer has a scrap value of $100 before it is disposed of. Find the number of hours of use before this occurs.

Common VCE mistakes:

Mixed units! One in dollars and the other in cents.

$100 = 850 - 0.25.n$

$n = 3000$

$V_n = 850 - 0.25.n$

$V_{50} = 850 - 0.25 \times 50$

$= \$837.50$

$425 = 850 - 0.25.n$

$n = 1700$

$\dfrac{1700}{17}$

$= 100$

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VCAA Questions

VCAA 2022 Further Maths
Exam 2

Question 6 (4 marks)

Pina owns workplace equipment, which she depreciates in value using flat rate depreciation.

The value of the equipment, in dollars, after n years, V_n, can be determined using the rule

$$V_n = 200\,000 - 12\,500n$$

a. Determine V_1, the value of the equipment after one year. 1 mark

$$V_1 = 200\,000 - 12\,500 \cdot 1$$

$$= \underline{\$187\,500}$$

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The value of the equipment, in dollars, after n years, V_n, can be determined using the rule

$$V_n = 200\,000 - 12\,500n$$

b. After how many years will the equipment first have a value of zero? 1 mark

$$0 = 200\,000 - 12500 \cdot n$$

$$n = 16 \text{ years}$$

VCAA Questions

VCAA 2020 Further Maths
Exam 1

Question 22

An asset is purchased for $2480.

The value of this asset after n time periods, V_n, can be determined using the rule

$$V_n = 2480 + 45n$$

A recurrence relation that also models the value of this asset after n time periods is

~~A.~~ $V_0 = 2480, \quad V_{n+1} = V_n + 45n$

~~B.~~ $V_n = 2480, \quad V_{n+1} = V_n + 45n$

C. $V_0 = 2480, \quad V_{n+1} = V_n + 45$

~~D.~~ $V_1 = 2480, \quad V_{n+1} = V_n + 45$

~~E.~~ $V_n = 2480, \quad V_{n+1} = V_n + 45$

$V_0 = 2480, \quad V_{n+1} = V_n + 45$

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